

# Chapter 1

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## Inequalities

### Learning Objectives :

After learning this chapter you will understand :

- **Linear Inequalities.**
- **Properties of Linear Inequalities.**
- **Solution of a Linear Inequality.**
- **Absolute Value.**
- **Linear Inequalities Involving Absolute Values.**

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**Basic Concepts**

1. **Linear Inequalities :** Let  $a$  and  $b$  be two real numbers. We say that  $a$  is greater than  $b$  (symbolically written as  $a > b$ ) if  $a - b > 0$ . Also we say that  $a$  is less than  $b$  (symbolically written as  $a < b$ ) if  $a - b < 0$ . Further we say that  $a \geq b$  if either  $a = b$  or  $a > b$ , and  $a \leq b$  if either  $a = b$  or  $a < b$ .
2. **Properties of Linear Inequalities :** Following are the some elementary properties of inequalities :
  - (i) Adding or subtracting the same number on both sides of an inequality does not affect the inequality, i.e., if  $a \leq b$  and  $c$  is any real number, then we have  $a \pm c \leq b \pm c$ .
  - (ii) The multiplication or division on both sides of an inequality by a positive number does not affect the inequality, i.e., if  $a \leq b$  and  $c$  is any real positive number, then we have  $a \times c \leq b \times c$ .
  - (iii) The multiplication on both sides by a negative number reverses the inequality, i.e., if  $a \leq b$  and  $c$  is any real negative number, then we have  $a \times c \geq b \times c$ .
  - (iv) The inequalities are transitive, i.e. if  $a \leq b$  and  $b \leq c$  then  $a \leq c$ .
  - (v) If  $0 < a < b$  then  $\frac{1}{b} < \frac{1}{a}$ .
  - (vi) If  $a \leq b$  and  $c \leq d$  then  $a + c \leq b + d$ .
3. **Solution of a Linear Inequality :** By solution of a linear inequality  $ax + b \leq c$  we mean all those values of  $x$  which satisfies the inequality  $ax + b \leq c$ .

**Exercise 1**

Solve the following inequalities

- |                           |   |  |
|---------------------------|---|--|
| Q1. $3x \leq 15$          | Q2. $3x < 15$                             | Q3. $8x - 2 > 5x$                              |
| Q4. $8x + 15 < 3x + 5$    | Q5. $\frac{4-2x}{3} \geq \frac{x}{2} - 3$ | Q6. $\frac{2x-3}{4} + 8 \geq 2 + \frac{4x}{3}$ |
| Q7. $\frac{x-3}{x+5} > 0$ | Q8. $\frac{x-5}{x+2} < 0$                 | Q9. $\frac{x+8}{x+2} > 1$                      |
- Q10. Solve  $13x < 200$ , when  
(i)  $x$  is a natural number                      (ii)  $x$  is an integer
- Q11. Find the values of  $x$  that satisfy the inequality :  
 $\frac{x}{\sqrt{x^2+1}} > \frac{x}{\sqrt{x^2-1}}$
- Q12. Find all  $x$  that satisfy the following inequalities :  
(i)  $\left| \frac{3x+1}{2} \right| < 1$                       (ii)  $\frac{x}{x^2+1} > \frac{x}{x^2-1}$
- Q13. Find all the real numbers  $x$  that satisfy the following inequalities :  
(i)  $\frac{1}{x} < x^2$ ,                      (ii)  $1 < x^2 < 4$ .



Q14. Find the solution set for the following inequalities :

(i)  $\frac{5}{x} > x - 4$

(ii)  $\sqrt{x+4} + \frac{1}{\sqrt{x-3}} \geq \sqrt{3-x}$

Q15. If  $x > \sqrt{xy} > y$ , deduce that  $x > y > 0$ .

Q16. Solve the inequalities

(i)  $\left|1 + \frac{3}{x}\right| > 2$

(ii)  $\frac{2}{x} < 3$

**Answers of Exercise 1**

1.  $x \leq 5$ ,      2.  $x < 5$ ,      3.  $x > 2/3$ ,      4.  $x < -2$ ,  
5.  $x \leq 26/7$ ,      6.  $x \leq 6.3$ ,      7.  $x > 3$  or  $x < -5$ ,      8.  $-2 < x < 5$ ,  
9.  $x > -2$ ,      10.(i)  $x \leq 15$ ,      (ii)  $x < \frac{200}{13}$ ,      11.  $-1 < x < 1$ ,

**Basic Concepts**

1. **Absolute Value :** If  $x$  is any real number, then its absolute value, denoted by  $|x|$ , is defined as follows :

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

2. **Linear Inequalities Involving Absolute Values :** The following are some special cases where linear inequalities involve absolute values :

If  $x$  is any real number and  $a$  and  $b$  are any positive real numbers

- (i)  $|x| < a \iff -a < x < a$   
(ii)  $|x| \leq a \iff -a \leq x \leq a$   
(iii)  $|x - a| < b \iff a - b < x < a + b$   
(iv)  $|x - a| \leq b \iff a - b \leq x \leq a + b$   
(v)  $|x| > a \iff x > a$  or  $x < -a$   
(vi)  $|x| \geq a \iff x \geq a$  or  $x \leq -a$   
(vii)  $|x - a| > b \iff x > a + b$  or  $x < a - b$   
(viii)  $|x - a| \geq b \iff x \geq a + b$  or  $x \leq a - b$

**Exercise 2**

Solve the following inequalities

- Q1.  $|x| < 4$       Q2.  $|2x - 3| < 5$       Q3.  $|3x - 7| > 2$   
Q4.  $|4 - x| + 1 < 3$       Q5.  $|7 - 3x| \leq 2$       Q6.  $\frac{2}{|x-3|} > 5$   
Q7.  $\left|\frac{3x-4}{2}\right| \leq \frac{5}{12}$       Q8.  $\frac{|3x+1|}{2} < 1$   
Q9. Solve the equation  $|7x - 5| = 0$  for  $x$ .



**Answers of Exercise 2**

- |    |                            |    |                      |    |                       |
|----|----------------------------|----|----------------------|----|-----------------------|
| 1. | $-4 < x < 4,$              | 2. | $-1 < x < 4,$        | 3. | $x > 3$ or $x < 5/3,$ |
| 4. | $2 < x < 6,$               | 5. | $5/3 \leq x \leq 3,$ | 6. | $13/5 < x < 17/5,$    |
| 7. | $19/18 \leq x \leq 29/18,$ | 8. | $-1 < x < 1/3$       |    |                       |

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