

# Chapter 2

## Measures of Central Tendency

### Learning Objectives :

After learning this chapter you will understand :

- Arithmetic Mean.
  - ✓ Weighted Arithmetic Mean,
  - ✓ Corrected Mean,
  - ✓ Combined Mean,
  - ✓ Applications of Arithmetic Mean.
- Geometric Mean.
- Harmonic Mean.
- Median.
- Trimmed Mean.
- Mode.
- Empirical Relation Between Mean, Median & Mode.
- Partition Values.

For Full Course Video Lectures of  
ALL Subjects of Eco (Hons), B Com (H), BBE & MA Economics  
Register yourself at  
[www.primeacademy.in](http://www.primeacademy.in)

**Dheeraj Suri Classes**

**Prime Academy**

+91 9899192027



**Basic Concepts**

**1. Meaning of Central Tendency.**

**Ans.** Central tendency of a data means an average figure of data that represents and describes the characteristic of the entire mass of unwieldy data. It may be defined as that value which is the most representative of typical of a group of data. Such a figure of central tendency depicts the characteristic of the whole group. It is also called average.

The main purposes served by central tendency figure is to get one single figure which describes and represents the entire data. It also facilitates comparison.

According to Croxton and Cowden, *“An average is a single value within the range of the data that is used to represent all the values in series. Since an average is somewhere within the range of the data, it is sometimes called a measure of central value.”*

Spurr, Kellog and Smith have defined it as *“An average is sometimes called a measure of central tendency because individual values of the variable usually cluster around it”*.

According to Clark, *“An average is a figure that represents the whole group.”*

**2. What are the types of averages ?**

**Ans.** In broad sense average can be divided into three kinds:

- (i). Mathematical Averages,
- (ii). Positional Averages,
- (iii). Special Averages

(i) **Mathematical Averages:** It can further be divided into three types:

- (a) **Arithmetic Average or Mean :** This is the simplest average. For individual observations, it is calculated by dividing the sum of observations by the number of observations.
- (b) **Geometric Mean :** It is the  $n$ th root of the product of ‘ $n$ ’ observations.
- (c) **Harmonic Mean :** It is the inverse of the arithmetic mean of the inverse of the values of the variable.

(ii) **Positional Averages:** It can further be divided into two types:

- (a) **Median :** This is the item which divides the distribution in such a way that equal number of items are on either side of it.
- (b) **Mode :** This is the most frequently occurring value of the variable.

(iii) **Special Averages :** These are of following two types :

- (a) Moving Average,
- (b) Progressive Average,
- (c) Composite Average,
- (d) Specialized Averages (e.g., index numbers).



3. Write the main features of a good statistical average.

Ans. The features of a good statistical average are:

- (i) **It should be rigidly and unambiguously defined**, *i.e.*, the answer should not change due to change in the method or formula of computation. The definition should be clear and un-ambiguous so that it leads to one and only one interpretation by different persons.
- (ii) **It should be easily understood**. The properties of the average should be such that they can be easily understood by persons of ordinary intelligence.
- (iii) **It should be simple to compute**. An average should be calculable with reasonable ease and rapidity only then it can be widely used. It should not involve heavy arithmetic calculations.
- (iv) **It should be based on all observations of the series**. If it is not based on all the observations, it cannot be said to be the representative of the whole group.
- (v) **It should be free from undue influence of any one or group of items**.
- (vi) **It should have sampling stability**. It means that if one takes different samples, from the same population, the average of any sample should approximately turn out to be the same as those of other samples. The difference in the averages in the two samples is technically called '*fluctuation of samples*'.
- (vii) **It should be capable of further use in statistical computations**, *e.g.*, computation of combined average etc. In other words, the average should possess some important mathematical properties so that its use in further statistical theory is enhanced otherwise its use is bound to be very limited.
- (viii) **It should be least affected by extreme observations**. Extreme observations mean some very large or very small values. Thus some small or large values should not unduly affect the good average.
- (ix) **It should be certain**. A good average must be certain in character. Only then an average value can be used as the basis of statistical analysis.
- (x) **It should be an absolute number**. A good average should be an absolute number. A percentage or a relative value does not serve as a good average.

#### Arithmetic Mean

1. What is arithmetic mean ?

Ans. Arithmetic average is the most popular and best undertaken measure of central tendency for a quantitative set of data. The arithmetic mean of a series is calculated by adding the values of all the scores of the series and divide the sum total by the number of scores. It is denoted by  $\bar{X}$  which is read as 'X bar'.

2. What are the types of arithmetic mean ?

Ans. **Types of Arithmetic Average** : There are two types of arithmetic averages



- (a). **Simple Arithmetic Average:** In this all the items are treated alike for calculating averages.
- (b). **Weighted Arithmetic Average:** In this the items are assigned weights and then the averages are calculated.
3. **Population and Sample Mean :** The average of all the values in a population is called population mean and it is denoted by  $\mu$ . Whereas its point estimate (or best guess) is the sample mean which represents the average of all the values in a sample. It is denoted by  $\bar{X}$ .
4. **List of formulae to calculate the Arithmetic Mean :**

	Individual Series	Discrete Series	Continuous Series
Direct Method	$\bar{X} = \frac{\sum_{i=1}^n x_i}{n}$	$\bar{X} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$	$\bar{X} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$
Short Cut Method (Assumed Mean Method)	$\bar{X} = A + \frac{\sum_{i=1}^n d_i}{n}$	$\bar{X} = A + \frac{\sum_{i=1}^n f_i d_i}{\sum_{i=1}^n f_i}$	$\bar{X} = A + \frac{\sum_{i=1}^n f_i d_i}{\sum_{i=1}^n f_i}$
Step deviation Method	$\bar{X} = A + \frac{\sum_{i=1}^n u_i}{n} \times h$	$\bar{X} = A + \frac{\sum_{i=1}^n f_i u_i}{\sum_{i=1}^n f_i} \times h$	$\bar{X} = A + \frac{\sum_{i=1}^n f_i u_i}{\sum_{i=1}^n f_i} \times h$

Where,  $d_i = x_i - A$ ,  $u_i = \frac{x_i - A}{h}$ ,

A is the assumed mean, and  $h$  is the class size

5. **Properties of Arithmetic Mean :**
- (i) The algebraic sum of the deviations from arithmetic mean (taking signs into account) is zero, i.e.  $\sum_{i=1}^n (x_i - \bar{x}) = 0$ . Arithmetic mean is a point of balance and the sum of the positive deviations is equal to the sum of the negative deviations.
- (ii) The sum of the squares of the deviations of a set of values is minimum when taken about mean, i.e.  $\sum (X - \bar{X})^2$  is minimum.
- (iii) Arithmetic mean is dependent of change of origin, i.e., if each observation is increased or decreased by a scalar 'a' then the mean also gets increased or decreased by 'a'.
- (iv) Arithmetic mean is dependent of change of scale, i.e. if each observation is multiplied or divided by a scalar 'a' then the mean also gets multiplied or divided by 'a'.



- (v) Arithmetic mean is capable of further arithmetic treatment, i.e., the formula of arithmetic mean can be extended to compute the combined average of two or more related series.
- (vi) If the two variables X and Y are linearly related, i.e.,  $Y = bX + c$ , then their mean are also linearly related  $\bar{Y} = b\bar{X} + c$ .
- (vii) The standard error of arithmetic mean is less than that of any other measure of central tendency.
- (viii) If we replace each item in the series by the mean, then the sum of these substitutions will be equal to the sum of the individual items.

**6. Write down the merits of arithmetic mean.**

**Ans.** The merits of arithmetic mean are given as under :

- (i) It is easy to understand, even a layman can understand it.
- (ii) It is simple to calculate because neither arrangement of data nor grouping of data is not required to compute mean. Therefore, it is commonly used method.
- (iii) It is based on all the items of the series. Therefore it represents the whole series.
- (iv) It is rigidly defined, i.e., its value is always definite. It means that if different formulas are used to compute the mean the answer will be the same.
- (v) It is capable of further algebraic treatment so it can be used in further statistical analysis.
- (vi) It is least affected by sampling fluctuations, i.e., it has sampling stability.
- (vii) Its computation does not require arrangement of elements, since it is not a positional average. So it is a computed value and not a positional value like Median or Mode.
- (viii) The mean is typical in the sense that it is the centre of gravity balancing the values on either side of it.
- (ix) It is a good base for comparison.

**7. Write down the de-merits of arithmetic mean.**

**Ans.** The de-merits of arithmetic mean are given as under :

- (i) It is affected by extreme observations.
- (ii) It cannot be computed in the absence of one of the items. It can not be computed accurately for open ended distributions.
- (iii) For a U-shaped distribution (which has very high frequency at the beginning and at the end and low frequency at the middle) it is not the central value around which other values are concentrated.
- (iv) It is not useful for studying qualitative phenomena, e.g., beauty, honesty, intelligence, etc.



- (v) Arithmetic mean sometimes gives absurd results which cannot possibly exist. e.g., average persons in a family 3.4 or 4.5 etc, which is not an item in the series therefore it is called fictions average.
- (vi) It cannot be decided only by observation, it requires actual calculations.

**8. Weighed Arithmetic Mean :**

**Ans.** In simple arithmetic mean all items are given equal importance. However, at times we face data and items where all units are not equally important. In such a case we use weighted mean. The term weight stands for the relative importance of the different items of the series. In weighted mean weights are attached to different items on the basis of their significance, e.g., expenditure on food is more important than expenditure on entertainment, so the weight of food expenditure should be more than the weight of entertainment expenditure. If  $x_1, x_2, x_3, \dots, x_n$  are the values of the variable and their relative weights are  $w_1, w_2, w_3, \dots, w_n$  respectively, then their average is known as weighted mean.

If  $w_1, w_2, w_3, \dots, w_n$  are weights assigned to the values  $x_1, x_2, x_3, \dots, x_n$  respectively, then the weighted arithmetic mean is calculated as under :

$$\bar{X}_w = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}$$

**9. Corrected Mean :**

If some observations are wrongly taken in the data then the corrected mean is

$$\text{calculated as under : } \text{Corrected Mean} = \frac{\text{Incorrect } \sum x - \text{Incorrect item}(s) + \text{Correct item}(s)}{n}$$

**10. Combined Mean :**

The combined mean of two or more different series may be calculated by the formula given below :

$$\bar{X}_{12} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

where,  $\bar{X}_{12}$  is the combined mean of the two series,

$\bar{x}_1$  is the mean of the first series,

$\bar{x}_2$  is the mean of the second series,

$n_1$  number of observations in the first series, and

$n_2$  number of observations in the second series.

**Exercise 1**

Q1. Find the Arithmetic mean of the numbers :

1, 2, 3, ..., n.



- Q2. Find arithmetic mean of the following data :  
5    20    10    15    25
- Q3. Find arithmetic mean of the following data :  
X :    5    10    15    20    25  
f :    7    6    9    10    12
- Q4. Find arithmetic mean of the following data :  
X :    10    20    30    40    50  
f :    2    5    8    20    10
- Q5. Find arithmetic mean of the following data :  
CI :    0-10    10-20    20-30    30-40    40-50    50-60    60-70  
f :    8    6    10    20    5    12    10
- Q6. From the following data calculate mean by  
(i) Direct method, (ii) Shortcut method, and (iii) Step deviation method  
Marks :    0-10    10-20    20-30    30-40    40-50  
No. of Students :    20    24    40    36    20
- Q7. Calculate the mean of the following frequency distribution relating to the marks secured by students in Statistics :  
Marks :    0-5    5-10    10-15    15-20    20-25    25-30    30-35    35-40    40-45  
No. of students :    1    6    8    7    11    10    10    17    20  
Marks :    45-50    50-55    55-60    60-65    65-70    70-75    75-80  
No. of students :    25    12    7    6    5    4    1
- Q8. Calculate Mean for the following data :  
Class Intervals:    10-19    20-29    30-39    40-49    50-59    60-69    70-79    80-89  
Frequencies :    32    42    40    56    20    6    2    2
- Q9. The mean of 50 observations is 25 if each observation is divided by 5, find the new mean.
- Q10. The mean of 100 observations is 30. If each observation is increased by 5, find the new mean.
- Q11. The algebraic sum of the deviation of 20 observations measured from 30 is 2. The mean of those observations is.....
- Q12. Which average would you use in case of sale of shirts of sizes (in inches) :  
16, 15½, 15, 15, 14, 13, 15  
Justify your answer.

### Missing Frequencies

- Q13. The numbers 3.2, 5.8, 7.9 and 4.5 have frequencies  $x$ ,  $(x + 2)$ ,  $(x - 3)$  and  $(x + 6)$  respectively. If their arithmetic mean is 4.876, find the value of  $x$ .
- Q14. For the data given below, find the missing frequency if the Arithmetic Mean is 33. Also find the median of the series:  
Loss per Shop :    0-10    10-20    20-30    30-40    40-50    50-60  
No. of shops :    10    15    30    -    25    20
- Q15. The mean of the following frequency table is 50. But the frequencies  $f_1$  and  $f_2$  in classes 20 – 40 and 60 – 80 are missing. Find the missing frequencies.



Class	:	0–20	20–40	40–60	60–80	80–100	Total
Frequency	:	17	$f_1$	32	$f_2$	19	120

**Weighted Mean**

Q16. Calculate weighted mean from the following data :

Value	:	10	12	15	18	20
Weight	:	2	5	12	4	7

Q17. A candidate obtained the following percentage of marks in different subjects in the half yearly examination :

English	46 %	Economics	58 %
Statistics	67 %	Income Tax	53 %
Cost Accountancy	72 %		

It is agreed to give double weights to marks in English and Statistics as compared to other subjects. What is the simple and weighted arithmetic mean?

**Combined Mean**

Q18. The number of employees in two branches, say A and B of a company, are 80 and 65 respectively. Average salary of employees in branch A is Rs. 875 per month and in B is Rs. 1,260 per month. Give the formula and calculate the combined average salary of the two branches.

Q19. Find out combined mean from the following data :

	Series X	Series Y
Arithmetic Mean	12	20
Number of Items	80	60

Q20. The mean of marks in Statistics of 100 students of a class was 72. the mean of marks of boys was 75, while their number was 70. find out the mean marks of girls in the class.

Q21. A factory employs 100 workers of whom 60 work in first shift and 40 work in second shift. The average wage of all the 100 workers is Rs. 38. If the average wage of 60 workers of the first shift is Rs. 40, find the average wage of the remaining 40 workers of the second shift.

Q22. The average salary of male employees in a firm was Rs. 520 and that of females was Rs. 420. The mean salary of all the employees was Rs. 500. Find the percentage of male and female employees.

Q23. The mean annual salary of all employees in a company is Rs. 25,000. The mean salary of male and female employees is Rs. 27,000 and Rs. 17,000 respectively. Find the percentage of males and females employed by the company.

Q24. The mean age of a combined group of men and women is 25 years. If the mean age of the group of men is 26 and that of the group of women is 21 years, find out the percentage of men and women in the group.

Q25. The mean weight of 50 students in a certain class is 59 kilograms. The mean weight of the boys in the class is 65 kilograms and that of girls is 50 kilograms. Find the number of girls in the class.



- Q26. The mean marks got by 300 students in the subject of Statistics are 45. the mean of top 100 of them was found to be 70 and the mean of the last 100 was known to be 20. What is the mean of the remaining 100 students.
- Q27. B. Com. (P) III year has three sections A, B and C with 50, 40 and 60 students respectively. The mean marks for the three sections were determined as 85, 60 and 65 respectively. However, marks of a student of section A were wrongly recorded as 50 instead of zero. Determine the mean marks of all the three sections put together.
- Q28. Fifty students took up a test. The result of those who passed the test is given below :
- |                        |   |   |    |   |   |   |   |
|------------------------|---|---|----|---|---|---|---|
| <b>Marks</b>           | : | 4 | 5  | 6 | 7 | 8 | 9 |
| <b>No. of Students</b> | : | 8 | 10 | 9 | 6 | 4 | 3 |
- If the average for all the 50 students was 5.16 marks, find the average of those who failed.
- Q29. 100 students appeared for an examination. The results of those who failed are given below :
- |                        |   |   |    |    |    |    |    |              |
|------------------------|---|---|----|----|----|----|----|--------------|
| <b>Marks</b>           | : | 5 | 10 | 15 | 20 | 25 | 30 | <b>Total</b> |
| <b>No. of Students</b> | : | 4 | 6  | 8  | 7  | 3  | 2  | <b>30</b>    |
- If the average marks of all students were 68.6, find out average of those who passed.
- Q30. 100 students appeared in an examination. The result of 30 students who failed are given below :
- |                        |   |   |    |    |    |    |    |              |
|------------------------|---|---|----|----|----|----|----|--------------|
| <b>Marks</b>           | : | 5 | 10 | 15 | 20 | 25 | 30 | <b>Total</b> |
| <b>No. of Students</b> | : | 3 | 7  | 8  | 7  | 3  | 2  | <b>30</b>    |
- If the average marks of all students were 60, find out average of those who passed.
- Q31. Initially there were 9 workers, all being paid a uniform wage. Later a 10<sup>th</sup> worker is added whose wage rate is Rs. 20 less than for the others. Compute the effect on the mean wage. **[Eco (H) 1998]**
- Q32. The average rainfall for a week, excluding Sunday was 10 cm. Due to heavy rainfall on Sunday, the average for the week rose to 15 cm. How much rainfall was there on Sunday.
- Q33. X Ltd. Has 5 plants producing P.V.C. pipes. The mean monthly production of 4 plants for 1982 was 73,000 feet and that of the fifth plant was 85,000 feet. Calculate the average production of the five plants.

**Corrected A.M.**

- Q34. Mean value of the weekly income of 40 families is 265. But in the calculation, income of one family was read as 150 instead of 115. Find the corrected mean.
- Q35. The mean of 200 items was 50. Later on it was discovered that two items were misread as 92 and 8 instead of 192 and 88. Find the corrected mean.
- Q36. The daily sales of a store for a given month were Rs. 2,750. During the month, the highest and lowest sales were Rs. 8,950 and Rs. 580 respectively. Find the average daily sales if the highest and lowest sales are not taken into account.



- Q37. Average marks in Statistics of 10 students of a class was 68. A new student took admission with 72 marks whereas two existing students left the college. If the marks of these students were 40 and 39, find the average marks of the remaining students.
- Q38. The mean of a certain number of items is 20. If an observation 25 is added to the data, the mean becomes 21. Find the number of items in the original data.

**Miscellaneous Problems**

- Q39. The sum of deviations of a certain number of observations measured from 4 is 72 and the sum of deviations of the same value from 7 is  $-3$ . Find the number of observations and their mean.
- Q40. The mean of 99 items is 5. The value of 100<sup>th</sup> item is 99 more than the mean of 100 items. What is the value of 100<sup>th</sup> item?
- Q41. Find the class intervals if the arithmetic mean of the following distribution is 33 and assumed mean is 35 :
- |                 |   |    |    |    |    |    |    |
|-----------------|---|----|----|----|----|----|----|
| Step-deviations | : | -3 | -2 | -1 | 0  | 1  | 2  |
| Frequency       | : | 5  | 10 | 25 | 30 | 20 | 10 |
- Q42. In the following data,  $X$  are the mid values of the class intervals and  $C$  is a constant. If the arithmetic mean of the original distribution is 35.84, find its class intervals.
- |           |     |     |    |    |    |    |    |
|-----------|-----|-----|----|----|----|----|----|
| $X - C$ : | -21 | -14 | -7 | 0  | 7  | 14 | 21 |
| $f$ :     | 2   | 12  | 19 | 29 | 20 | 13 | 5  |

**Applications of A. M.**

- Q43. A person walks 9 hours at a speed of 3 kms per hour and again walks 8 hours at a speed of 4 kms per hour. Find the average speed per hour by using appropriate average.
- Q44. A person travels for 4 hours at the speed of 40 kms per hour and next 2 hours at the speed of 50 kms per hour. Find out his average speed.
- Q45. A scooterist purchased petrol at the rate of Rs. 14.00, Rs. 15.50 and Rs. 16.00 per litre during three successive years. Calculate the average price of petrol, if he purchased 150, 160 and 170 litres of petrol in three years.
- Q46. The rate of a certain commodity in the first week of January, 2002 is 0.4 kg per rupee. It is 0.6 kg per rupee in the second week and 0.5 kg in the third week. Therefore, it is correct to say that the average price is 0.5 kg per rupee. Verify.
- Q47. A man traveled by car for 3 days. He covered 480 kms each day. On the first day he drove for 10 hrs at 48 kms per hour. On the second day he drove for 12 hours at 40 kms an hour and on last day he drove for 15 hours at 32 kms per hour. What was his average speed ?
- Q48. Mr. Chopra bought 20 share of stock at Rs. 15 each, 50 shares of Rs. 20, 100 shares of Rs. 30 and 75 shares of Rs. 35.
- (i) What is the average price per share?
- (ii) What is the total amount of his investment?



**MA Entrance**

- Q49. Provided  $c \neq 0$ , the expression  $\frac{\sum_{i=1}^n (c - x_i) - \sum_{i=1}^n (cy_i - x_i)}{c}$  is identical to  
(a)  $1 - n\bar{y}$  (b)  $n - n\bar{y}$  (c)  $n + n\bar{y}$  (d)  $1 + n\bar{y}$   
**[DSE MA Ent. Eco. 2005]**
- Q50. Members of the Gymkhana club are charged quarterly fees on the basis of their average weight, rounded to the nearest kg. Of the 560 members, 120 weighted between 60 and 69 kg., 140 weighted between 70 and 79 kg., 170 weighted between 80 and 89 kg., and the remaining weighted between 90 and 99 kg. If members are charged Rs. 50 per kilo of their weight, on average how much each member pay ?  
**[DSE MA Ent. Eco. 2006]**  
(a) Rs. 3800 (b) Rs. 3900 (c) Rs. 4000 (d) Rs. 4100
- Q51. The arithmetic mean of a set of 10 numbers is 20. If each number is first multiplied by 2 and then increased by 5, then what is the mean of new numbers?  
(a) 20 (b) 25 (c) 40 (d) 45  
**[Ans. : (d)]**
- Q52. The mean of 25 observations is 36. The mean of first 13 observations is 32 and that of last 13 observations is 39. What is the value of 13th observation?  
(a) 20 (b) 23 (c) 32 (d) 40  
**[Ans. : (b)]**
- Q53. The average age of 06 persons living in a house is 23.5 years. Three of them are majors and their average age is 42 years. The difference in ages of the three minor children is same. What is the mean of the ages of minor children?  
(a) 3 years (b) 4 years (c) 5 years (d) 6 years  
**[Ans. : (c)]**
- Q54. What is the weighted mean of first 10 natural numbers whose weights are equal to the corresponding number?  
(a) 7 (b) 5.5 (c) 5 (d) 4.5  
**[Ans. : (a)]**
- Q55. In a class of 45 students a boy is ranked 20th. When two boys joined, his rank was dropped by one. What is his new rank from end?  
(a) 25<sup>th</sup> (b) 26<sup>th</sup> (c) 27<sup>th</sup> (d) 28<sup>th</sup>  
**[Ans. : (a)]**
- Q56. The mean age of combined group of men and women is 25 years. If the mean age of group of men is 26 and that of group of women is 21, then percentage of men and women in the group respectively is:  
(a) 60, 40 (b) 80, 20 (c) 30, 70 (d) 50, 50  
**[Ans. : (b)]**
- Q57. The arithmetic mean (average) of the first ten whole numbers is  
(a) 5.5 (b) 5 (c) 4 (d) 4.5  
**[Ans. : (c)]**



Q58. The mean of 9 observations is 16. One more observation is included and the new mean becomes 17. The 10th observation is

- (a) 18                      (b) 26                      (c) 30                      (d) 7

[Ans. : (c)]

**Answers of Exercise 1**

1.  $(n + 1)/2$ , 2. 15, 3. 16.59, 4. 39.11, 5. 36.83,  
6. 25.85, 7. 39.37, 8. Mean = 35.8, 9. Mean = 5, 10. Mean = 35,  
11. 30.1, 12. 14.79, 13. 0.2, 14. 33, 15.  $f_1 = 28$  &  $f_2 = 24$ ,  
16. 15.73, 17.  $\bar{x} = 59.2\%$ ,  $\bar{x}_w = 58.43\%$ , 18. 1047.59,  
19. Combined Mean = 15.43, 20. Mean of girls = 65,  
21. Mean wage of second shift = Rs. 35, 22. 80% males and 20% females,  
23. 80% males and 20% females, 24. 80% men and 20% women,  
25. 20, 26. 45, 27. 70, 28. 2.1, 29. 91.21, 30. 78.86,  
31. mean wage decreases by Rs. 2, 32. 45 cms,  
33. 75,400 feet, 34. 264.12, 35. 50.9, 36. Rs. 2,606, 37. 74.78,  
38. 4, 39. 25 and 6.88, 40. 105,  
41. 0-10, 10-20, 20-30, 30-40, 40-50, 50-60,  
42. 10.5-17.5, 17.5-24.5, 24.5-31.5, 31.5-38.5, 38.5-45.5, 45.5-52.5, 52.5-59.5,  
43. 3 km/hr., 44. 43.33 Km/Hr., 45. Rs. 15.21,  
46. No, Correct mean is 0.486kg per rupee, 47. 37.92 km/hr.  
48.(i) 28.27, (ii) 6925, 49. (b), 50. (c), 51. (d),  
52. (b), 53. (c), 54. (a), 55. (a), 56. (b),  
57. (c), 58. (c),

**Basic Concepts**

**Geometric Mean**

1. **Definition :**

If  $x_1, x_2, x_3, \dots, x_n$  are  $n$  values of a variate  $x$ , none of them being zero or negative, then the Geometric Mean is defined as under :

$$G.M. = \sqrt[n]{x_1 \times x_2 \times x_3 \times \dots \times x_n}$$

**Note :** If any of  $x_1, x_2, x_3, \dots, x_n$  is negative or zero then geometric mean can not be calculated.

Since it is difficult to find the  $n$ th root, therefore, geometric mean is calculated using the logarithms as under :

$$G.M. = \text{anti log} \left[ \frac{\sum_{i=1}^n \log x_i}{n} \right]$$



2. For grouped data the geometric mean is calculated by the following formula :

$$G.M. = \text{anti log} \left[ \frac{\sum_{i=1}^n f_i \log x_i}{\sum_{i=1}^n f_i} \right]$$

3. **Uses of geometric mean :**

It is used to calculate the average of rate of change of ratio's given in percentages.

4. **Weighted Geometric Mean :** Weighted geometric mean is calculated by using the following formula :

$$G.M._w = \text{anti log} \left[ \frac{\sum_{i=1}^n w_i \log x_i}{\sum_{i=1}^n w_i} \right]$$

5. **Write down the merits of geometric mean.**

**Ans.** The merits of geometric mean are as under :

- (a) All the observations are considered in G.M.
- (b) Algebraic manipulations are possible.
- (c) It is a precise result as it is defined rigidly.

6. **Write down the de-merits of geometric mean.**

**Ans.** The de-merits of geometric mean are as under :

- (a) Calculation is difficult and is not generally understood.
- (b) Geometric becomes imaginary, where an item is zero or negative.

7. **Mention the properties of geometric mean.**

**Ans.** The main properties of geometric mean are given as under :

- (a) The product of the values of the series will remain unchanged if the value of the geometric mean is substituted for each individual value. e.g. the geometric mean for the series 3, 6, 12 is 6, therefore we have  $3 \times 6 \times 12 = 216 = 6 \times 6 \times 6$ .
- (b) The sum of the deviations of the logarithms of the original observations above or below the logarithm of the geometric mean is equal. In other words, the value of the geometric mean balances the ratio deviations of the observations from it.

8. **Compound Interest Formula :**

The average annual percent increase may be computed by applying the following formula:

$$P_n = P_0(1 + r)^n$$



where,  $P_n$  is the value at the end of  $n$  periods,  
 $P_0$  is the initial value,  
 $r$  is the rate of change, and  
 $n$  is the number of periods.

**Exercise 2**

- Q1. Calculate the geometric mean of 3, 6, 24 and 48.  
Q2. Find out geometric mean of the following :  
1, 7, 18, 65, 91 and 103  
Q3. Find out geometric mean of the following :  
1, 7, 29, 92, 115 and 375  
Q4. Find the geometric mean of 2, 4, 8, 12, 16 and 24.  
Q5. Calculate the geometric mean of the following :  
2574, 475, 75, 5, 0.8

**Applications of Geometric Mean**

- Q6. A sum of money was invested for four years. The respective rates of return per annum were 4%, 5%, 6% and 8%. Determine the average rate of interest per annum.  
Q7. The amounts of interest paid of each of the three different sums of money yielding 10%, 12% and 15% simple interest per annum are equal. What is the average yield percent on the total sum invested?  
Q8. The population of a country increased by 20% in the first decade, 30% in second decade and 45% in the third decade. Find the average rate of increase in population per decade.  
Q9. The annual rates of growth achieved by a nation for 5 years are 5%, 7.5%, 2.5%, 5% and 10% respectively. What is the compound rate of growth for the 5 year period?  
Q10. The annual rates of growth of output of a factory in five years are 5.0, 6.5, 4.5, 8.5 and 7.5 percent respectively. What is the compound rate of growth of output per annum for the period.  
Q11. A business depreciated the machinery of his factory by 20% in the first two years and 40% in the third year. What is the average depreciation for the three years?  
Q12. A machinery is assumed to depreciate at the rate of 40% in the value in the first year, @ 25% in value in the second year and @ 10% per annum in value for the next three years, each percentage being calculated on diminishing value. What is the average percentage depreciation for the entire period?  
Q13. The value of a machine is depreciated by 30% in first year, 13% in second year and 5% in each of the following three years. Determine the average rate of depreciation for the entire period by using an appropriate measure of central value.  
Q14. A machine is assumed to depreciate at the rate of 35.5% p.a. in value in the first year, @ 22.5% p.a. in value in the second year and @ 9.5 % p.a. in value in the third year, each percentage being computed on the diminishing value. What is the average percentage depreciation for the three years?



- Q15. The cost of a machinery has appreciated by 10%, 12% and 8% in three years but in the fourth year it depreciated by 5%. Find out the average increase or decrease in the cost of the machinery during four years.
- Q16. The price of a machinery depreciates by 5% in first year, by 7% in second year, by 15% in third year but the price appreciates by 20% in fourth year. Find out the average price (increase or decrease) after four years.
- Q17. Geometric mean of 2 numbers is 5. If by mistake one figure is taken as 5, instead of 3, find correct geometric mean.
- Q18. The geometric mean of four values was calculated as 16. It was later discovered that one of the values was recorded wrongly as 32 when, in fact, it was 162. Calculate the correct geometric mean.
- Q19. A certain store made profits of Rs. 5,000, Rs. 10,000 and Rs. 80,000 in 1965, 1966 and 1967 respectively. Determine the average rate of the growth of this store's profits.
- Q20. A certain store's profits were Rs. 50,000, Rs. 1,00,000 and Rs. 8,00,000 in 2002, 2003 and 2004 respectively. Determine the average rate of growth of the store's profits.
- Q21. First the missing information in the following table:

	Groups			
	A	B	C	Combined
Number	10	8	-	24
Mean	20	-	6	15
Geometric Mean	10	7	-	8.397

### **Weighted Geometric Mean**

- Q22. The weighted geometric mean of three numbers 229, 275 and 125 is 203. The weights for the first and second numbers are 2 and 4 respectively. Find the weight of third.
- Q23. The Weighted Geometric Mean of 5 numbers - 10, 15, 25, 12 and 20, is 18.15. If the weights of the first four numbers are 2, 3, 5, and 2 respectively, find the weight of the 5<sup>th</sup> number.

### **Compound Interest Formula**

- Q24. The number of divorces per 1,000 marriages in a big city in India increased from 96 in 1980 to 120 in 1990. Find the annual rate of increase of the divorce rate for the period 1980 to 1990.
- Q25. A piece of property was purchased for Rs. 20 lakhs and sold for Rs. 32.60 lakhs after 10 years. What is the average annual rate of return on the original investment ?
- Q26. The number of bacteria in a certain culture was found to be  $4 \times 10^6$  at noon of one day. At noon the next day the number was  $9 \times 10^6$ . If the number increased at a constant rate per hour, how many bacteria were there at the intervening midnight?



- Q27. The population of a city was 1,00,000 in 1975 and 1,44,000 a decade later. Estimate the population at the middle of the decade 1975-85.
- Q28. In 1950 and 1960, the population of the US was 151.3 mn and 179.3 mn respectively.
- What was the average percentage increase per year?
  - Estimate the population in 1954
  - If the average percentage increase of population per year from 1960 to 1970 is the same as in (i), what would be the population in 1970?

**Answers of Exercise 2**

1. 12, 2. 20.61, 3. 30.5, 4. 8.158, 5. 51.63, 6. 5.7%,  
7. 12.3%, 8. 31.3%, 9. 5.9%, 10. 6.4%, 11. 27.31%, 12. 20%,  
13. 12.19%, 14. 23.23%, 15. Increase of 5.92%, 16. 2.57% decrease,  
17. 3.87, 18. 24, 19. 300%, 20. 300%,  
21.  $N_c = 6$ ,  $\bar{x}_B = 15.5$ ,  $GM_c = 8$ , 22. 3, 23. 10, 24. 2.21%,  
25. 5%, 26.  $r = 3.43\%$ ,  $6 \times 10^6$ , 27.  $r = 3.71\%$ , 1,19,978,  
28.(i)  $r = 1.71\%$ , (ii) 161.94 mn, (iii) 212.43 mn

**Basic Concepts**  
**Harmonic Mean**

1. **Definition :**

If  $x_1, x_2, x_3, \dots, x_n$  are  $n$  values of a variate  $x$ , none of them being zero, then the Harmonic Mean is defined as under :

$$H.M. = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}, \text{ i.e. } H.M. = \frac{n}{\sum_{i=1}^n \left( \frac{1}{x_i} \right)}$$

*Note :* If any of  $x_1, x_2, x_3, \dots, x_n$  is zero then Harmonic mean can not be calculated.

2. For grouped data the harmonic mean is calculated by the following formula :

$$H.M. = \frac{\sum_{i=1}^n f_i}{\sum_{i=1}^n \left( \frac{f_i}{x_i} \right)}$$

3. **Weighted Harmonic Mean :** Weighted harmonic mean is calculated by using the following formula :



$$H.M._w = \frac{\sum_{i=1}^n w_i}{\sum_{i=1}^n \left( \frac{w_i}{x_i} \right)}$$

**4. Uses of Harmonic Mean :**

It is used to find the average of rates, average of speeds, average price per unit etc.

**5. Relation between AM, GM and HM**

Let a and b be two numbers then  $AM = \frac{a+b}{2}$ ,  $GM = \sqrt{a \times b}$ ,  $HM = \frac{2ab}{a+b}$

**6. Distinguish between Geometric mean and Harmonic mean.**

- Ans.** (i) Geometric mean is based on an arithmetic mean of logarithm while Harmonic mean is based on arithmetic mean of reciprocals.  
 (ii) Geometric mean is a calculated value and depends upon the size of all the items where as Harmonic mean is difficult to compute and is not understandable to the common man.  
 (iii) Geometric mean exists only for positive value of the variate while the Harmonic mean is useful in the averaging of time rates and other similar phenomenon.

**7. Compare Arithmetic, Geometric and Harmonic Mean.**

**Ans.** The various features of arithmetic mean, geometric mean and harmonic mean are compared in the following table :

S. No.		Arithmetic Mean	Geometric Mean	Harmonic Mean
1.	Calculations takes into account	All values	All values	All values
2.	Effect of extreme values	Greatest effect	Effect less than A.M. but more than H.M.	Least effect
3.	Value	Greatest value	Value less than A.M. but more than H.M.	Minimum Value
4.	Simplicity	Extremely simple, even a layman can understand its computation.	Involves very complex statistical calculations.	Its complexity is moderate.
5.	If one value zero	Can be calculated	Cannot be calculated	Cannot be calculated



6.	If one value negative	Can be calculated	Cannot be calculated	Can be calculated
----	-----------------------	-------------------	----------------------	-------------------

### **Exercise 3**

Q1. Find harmonic mean of the following data :

5      7      10      15      25

Q2. Find harmonic mean of the following data :

40      100      2      50      8

Q3. Find harmonic mean of the following data :

X :      5      10      15      20      25

f :      7      6      9      10      12

Q4. Find harmonic mean of the following data :

X :      10      20      30      40      50

f :      2      5      8      20      10

Q5. Find harmonic mean of the following data :

CI :      0-10      10-20      20-30      30-40      40-50      50-60      60-70

f :      8      6      10      20      5      12      10

### **Application of Harmonic Mean**

Q6. Mr. X travels from Delhi to Ghaziabad at a speed of 40 km per hour, but while returning his speed was 60 km per hour. Find his average speed.

Q7. In a factory outlet a unit of work is completed by A in 10 minutes, by B in 15 minutes, by C in 16 minutes and by D in 20 minutes. What is the average number of units of work completed per minute?

Q8. In a certain factory, a unit of work is completed by A in 4 minutes, by B in 5 minutes, by C in 6 minutes, by D in 10 minutes and by E in 12 minutes. What is their average rate of working? What is the average number of units of work completed per minute? At this rate, how many units of work each one of them on the average will complete in a six-hour day?

Q9. Four factories emit a kilogram of pollutant each in 4, 5, 8 and 12 days respectively. What is the average rate of pollutant discharge? Use your answer to calculate the total pollutant discharged by the four factories in one week.

Q10. In a certain office a letter is typed by A in 4 minutes. The same letter is typed by B, C and D in 5, 6, 10 minutes respectively. What is the average time taken in completing one letter? How many letters do you expect to be typed in one day comprising of 8 working hours?

Q11. A person purchases one kilogram of cabbages from each of the four places at the rate of 20 kg, 16 kg, 12 kg and 10 kg per rupee respectively. On the average how many kg of cabbages has he purchased per rupee ?

Q12. Four typists take 15, 10, 6 and 5 minutes respectively to type a letter. Determine the average time required to type a letter by using 'harmonic mean' if :

(i) four letters are to be typed by each typist.



- (ii) each typist works for two hours.
- Q13. If X travels 8 km at 4 km per hour, 6 km at 3 km per hour and 4 km at 2 km per hour, what would be the average rate per hour at which he traveled?
- Q14. A motorist took 12 hours to travel from place A to place B which are situated 300 kms apart. On the return journey he took 15 hours to cover this distance. What is the average speed of the motorist ?
- Q15. A vehicle when climbing up a gradient, consumes petrol at the rate of 1litre per 8kms, while coming down it gives 12 kms per litre. Find its average consumption for to and for travel between two places situated at the two ends of a 25 kms long gradient. Verify your answer.
- Q16. A radio-taxi runs first 25 kilometers at a speed of 30 kilometers per hour, next 50 kilometers at a speed of 40 kilometers per hour, then due to repair of the road, it covers only one km at a speed of 10 kilometers per hour, and finally covers the remaining distance of 24 kilometers at a speed of 24 kilometers per hour to reach the destination. What is the average speed in kilometers per hour? Explain the choice of average.
- Q17. A train runs 25 miles at a speed of 30 m.p.h., another 50 miles at a speed of 40 m.p.h., then due to repairs of the track, travels 6 miles at a speed of 10 m.p.h., and finally covers the remaining distance of 24 miles at a speed of 30 m.p.h.. What is the average speed in miles per hour?
- Q18. A train runs for 30 minutes at a speed of 40 miles an hour and then because of repairs of the track, the train runs for 10 minutes at a speed of 8 miles an hour, after which it resumes its previous speed and runs for 20 minutes except for a period of two minutes when it had to run over a bridge with a speed of 30 miles per hour. Using a suitable average calculate the average speed of the train.
- Q19. You can take a trip which entails traveling 900 km. by train at an average speed of 60 km. per hour, 3000 km. by boat at an average speed of 25 km. per hour, 400 km by plane at an average speed of 350 km. per hour and finally 15 km. by taxi at an average speed of 25 km. per hour. What is your average speed for the entire trip.
- Q20. An airplane travels distance of 2,500, 1,200 and 500 kms at speeds 500, 400 and 250 kms per hour respectively. Find the average speed for the entire trip, commenting on the choice of your average.
- Q21. Cities A, B and C are equidistant from each other. A motorist travels from A to B at 30 km per hour from B to C at 40 kms per hour and from C to A at 50 kms per hour. Determine his average speed for the entire trip.
- Q22. If you spend Rs. 100 per week on apples and the price of apples for three weeks is Rs. 25, Rs.20 and Rs. 10 per kilogram, what is the average price of apples for you?
- Q23. An investor buys Rs. 1500 worth of shares in a company each month. During the first four months he bought the shares at a price of Rs. 10, Rs. 15, Rs. 20 and Rs. 30 per share. What is the average price paid for the shares by him ? Verify your result.



- Q24. A scooterist purchased petrol at the rate of Rs. 14.00, Rs. 15.50 and Rs. 16.00 per litre during three successive years. Calculate the average price of petrol.
- (i) If he purchased 150, 160 and 170 litres of petrol in three years and
  - (ii) If he spent Rs. 2200, Rs. 2,500 and Rs. 2,600 in the three years.
- Q25. A scooterist purchased petrol at the rate of Rs. 24.00, Rs. 29.50 and Rs. 36.85 per litre during three successive years. Calculate the average price of petrol.
- (i) If he purchased 150, 180 and 195 litres of petrol in the respective years; and
  - (ii) If he spent Rs. 3,850, Rs. 4,675 and Rs. 5,825 in the three years. Give support to your answer.
- Q26. During a period of decline in stock market prices, a stock sold at Rs. 50 per share on one day, Rs. 40 the next day and Rs. 25 on the third.
- (i) If an investor bought 100, 120 and 180 shares on the respective three days, find the average price paid per share.
  - (ii) If an investor bought Rs. 1000 worth of shares on each of the three days, find the average price paid per share.
- Q27. An investor buys Rs. 1200 worth of shares in a company each month. During the first five months he bought the shares at a price of Rs. 10, Rs. 12, Rs. 15, Rs. 20 and Rs. 24 per share. After 5 months what is the average price paid for the shares by him ?
- Q28. An aeroplane travels distances of  $d_1$ ,  $d_2$  and  $d_3$  km at speeds of  $V_1$ ,  $V_2$ , and  $V_3$  km. per hour respectively. Show that the average speed is given by  $V$ , where

$$\frac{d_1 + d_2 + d_3}{V} = \frac{d_1}{V_1} + \frac{d_2}{V_2} + \frac{d_3}{V_3}.$$

### **Relation between AM, GM and HM**

- Q29. If arithmetic mean and geometric mean of two values are 10 and 8 respectively, find values.
- Q30. If the arithmetic means of two items of a series is 12.5 and Geometric Mean is 10, what is the difference of two items ?
- Q31. If A is the arithmetic mean, H is the harmonic mean and G is the geometric mean of the two numbers, show that  $AH = G^2$ .
- Q32. If the A. M. of two numbers is 17 and G.M is 15, find the H.M. of these numbers.
- Q33. The AM and GM of the two numbers are 15 and 12 respectively. Find HM of the numbers.
- Q34. The geometric mean and arithmetic mean of two observations are 60 and 127.5 respectively. Find their harmonic mean. Also obtain the observations.
- Q35. If for series the arithmetic mean is 25 and the harmonic mean is 9, what is the geometric mean of the series?

### **Answers of Exercise 3**

1. 9.1,      2. 7.35,      3. 12.29,      4. 31.76,      5. 20.55,



6. 48 km/hr, 7. 0.07 units per minute,  
8. 6.25 minutes, 0.16 per minute, 57.6 units,  
9. 6.08 days per kg, 4.61 kg, 10. 5.58 minutes, 344 letters,  
11. 13.52 kgs. Per Rupee, 12.(i) 9 minutes, (ii) 7.5 minutes,  
13. 3 km/hr., 14. 22.22 km/hr., 15. 9.6 kms/litre, 16. 31.41 km/hr.,  
17. 30.14 m.p.h., 18. 34.33 miles per hour, 19. 31.56 km/hr, 20. 420 km/hr,  
21. 38.28 km/h, 22. Rs. 15.79, 23. Rs. 16, 24.(i) Rs. 15.21, (ii) Rs. 15.18,  
25.(i) Rs. 30.65 per litre, (ii) Rs. 30.09 per litre, 26.(i) Rs. 35.75, (ii) Rs. 35.29,  
27. Rs. 14.63, 29. 16 & 4, 30. 15, 32. 13.24, 33. 9.6,  
34. HM = 28.23, a = 240, b = 15, 35. 15

### Basic Concepts

#### Median

#### 1. Definition of Median.

Median is centrally located value of a series such that half of the values are above it and half are below it. It is for this reason median is called *Positional Average*. The term position refers to the place of a value in the series. The place of the median is such that equal number of items lie on either side of it. As you have studied above the change in the value of a single item could change the arithmetic mean drastically, whereas such change would generally have no impact on the median. It is denoted by Med or  $M_d$ . Median may also be defined as the middlemost value of the distribution, or as the value such that greater and smaller values occur with equal frequencies. If the number of items are odd then the middlemost value represents the median but if the number of items are even then median is indeterminate, in such a case it is arbitrarily taken as the mean of the two middle most values.

According to *Connor*, "The median is that value of the variable, which divides the group into two equal parts. One part comprising all values greater and the others all values less than the median".

According to *Yule and Kendal*, "The median may be defined as the middlemost value of the variable when the items are arranged in order of magnitude, or as the value such that greater and smaller values occur with equal frequency. In the case of a frequency curve the median may be defined as that value of the variable which divides the area of the curve into two equal parts".

According to *A. L. Bowley*, "If the number of the group are ranked in order according to the measurement under consideration then the measurement of the number most nearly one half is the median."

#### 2. Calculation of Median

**Individual series :** For Individual series first of all given data is arranged either in ascending order or in descending order and then median is calculated by the following formula :



- (i) If 'n' is odd then median =  $\left(\frac{n+1}{2}\right)^{th}$  observation
- (ii) If 'n' is even then median =  $\left(\frac{\left(\frac{n}{2}\right) + \left(\frac{n}{2} + 1\right)}{2}\right)^{th}$  observation, where 'n' represents the number of observations.

**Discrete Series :** For discrete series median is calculated by the following formula :

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

**Continuous Series :** For continuous series median is calculated by using the formula given as under :

$$\text{Med.} = L_1 + \frac{\frac{n}{2} - c.f.}{f} \times i$$

where,  $L_1$  is the lower limit of median class,

c.f. is the cumulative frequency of class preceeding the median class,

f is the frequency of median class, and

i is the class size.

**3. Properties of Median :**

- (i) The sum of the absolute deviations from the median is minimum, i.e.,

$$\sum |x_i - M_d| \text{ is minimum.}$$

- (ii) Median is the average of position and therefore is influenced by the position of items in arrangement and not by the size of the items.

- (iii) If x and y are two variables, to be related by  $y = a + bx$  for any two constants a and b, then the median of y is given by  $y_{md} = a + bx_{md}$ .

**4. Write down the merits of median.**

**Ans.** The merits of median are given as under :

- (i) Median is useful in case of open-ended series such as income distribution since median is based on the position and not on the value of items. It is useful in a distribution of unequal classes.
- (ii) Median is easier to compute than mean in case of unequal class-intervals and it is easy to understand.
- (iii) It is not affected by extreme values.
- (iv) It is most suitable average for dealing with qualitative data.
- (v) Besides algebraic computation, it can also be determined graphically.
- (vi) It minimizes the total absolute deviations.
- (vii) It is proper average for qualitative data where items are not converted or measured but are scored.
- (viii) Median value is always a certain value in the series.



- (ix) Median value is a real value and is a better representative value of the series compared to arithmetic average, the value of which may not exist in the series at all.
- (x) Median can be estimated even in the case of certain incomplete series. It is enough if one knows the number of items and the middle item(s) of the series.
- (xi) It is useful in measuring skewness and kurtosis.

**5. Write down the de-merits of median.**

**Ans.** The de-merits of median are given as under :

- (i) It requires the data in ascending/descending order.
- (ii) It is not based on all the items of the series.
- (iii) It is not capable of algebraic treatment.
- (iv) It is affected more by sampling fluctuations than the value of mean.
- (v) It is not accurate when the data is not large.
- (vi) Interpolation by a formula is required to calculate median in continuous series. This requires the assumption that all the frequencies of the class interval are uniformly spread which is not always true.
- (vii) Median fails to be a representative measure in case of such series the different values of which are wide apart from each other.
- (viii) When the median is located somewhere between the two middle values, it remains only an approximate measure, not a precise value.

**6. Explain the determination of median graphically.**

**Ans.** Median can be determined graphically using the following steps :

- (i) Calculate ascending cumulative frequencies (less than) and descending cumulative frequencies (more than).
- (ii) Draw two *ogives*, one by less than and other by more than methods.
- (iii) From the point of intersection of two ogives draw a perpendicular on  $x$ -axis.
- (iv) The point on  $x$ -axis where this perpendicular intersects is median.

7. **Symmetric Population :** If population mean  $\mu$  and population median  $\tilde{\mu}$  are equal the population distribution is symmetric. Whereas if population mean  $\mu$  is greater than population median  $\tilde{\mu}$  then population is positively skewed and if population mean  $\mu$  is less than population median  $\tilde{\mu}$  then population is negatively skewed.

8. **Trimmed Mean :** A trimmed mean is a compromise between  $\bar{X}$  (mean) and  $\tilde{X}$  median. If the desired trimming percentage is  $100\alpha\%$  and  $n\alpha$  is an integer then  $n\alpha$  smallest values and  $n\alpha$  largest values are eliminated. From a sample of 20 observations a 10% trimmed mean, for example would be computed by eliminating the smallest 2 and the largest 2 values of the sample and then averaging what is left



over. Whereas, If the desired trimming percentage is  $100\alpha\%$  and  $n\alpha$  is not an integer then the trimmed mean must be calculated by interpolation.

A trimmed mean with a moderate trimming percentage, say between 5% to 25%, will yield a measure of center that is neither as sensitive to outliers as is the mean and nor as insensitive as is the median.

### **Exercise 4**

- Q1. Find the median of the following data :  
17, 32, 35, 33, 15, 21, 41, 32, 11, 10, 20.
- Q2. On a particular warm day six shops reported early morning sales of cold drinks as follows :  
Shop :      A      B      C      D      E      F  
Sales :      73      80      36      75      68      82  
Calculate appropriate measure of central tendency and explain the reason for your choice.
- Q3. The heights in cms. of 30 persons are given below :  
133   125   137   129   130   130   131   125   137   147  
128   127   147   141   148   149   145   148   130   125  
145   134   129   145   127   147   132   128   130   131  
Prepare the frequency distribution table and find the median.
- Q4. Find the median of the following data :  
Price :      20      25      30      35      40      45      50      55  
 $f$  :      14      18      33      30      20      15      13      7
- Q5. Compute median of the following data :  
 $X$  :      5      10      15      20      25      30      35  
 $f$  :      7      9      12      10      5      6      15
- Q6. Find median value for the following data :  
No. of Children :      0      1      2      3      4      5      6  
No. of Families :      150      72      50      28      12      8      5
- Q7. Calculate mean and median of the following series :  
Weekly Wages :      100-105      105-110      110-115      115-120      120-125      125-130  
No. of workers:      200      210      230      320      350      520  
Weekly Wages :      130-135      135-140      140-145      145-150      150-155      155-160  
No. of workers:      410      320      280      210      160      90
- Q8. Given below is the distribution of marks obtained by 140 students in an examination:  
Marks :      10-19      20-29      30-39      40-49      50-59      60-69      70-79      80-89      90-99  
No. of Students :      7      15      18      25      30      20      16      7      2  
Find the Median and Mode of the distribution.
- Q9. Find out median from the following data :  
Wages per week :      50-59      60-69      70-79      80-89      90-99      100-109      110-119  
No. of Workers :      15      40      50      60      45      40      15



Q10. Find out the value of median from the following data :

Weight in lbs. : 60-64 65-74 75-79 80-84 85-94 95-99 100-104

No. of Students : 10 39 35 20 26 9 6

Q11. From the following distribution of marks in law obtained by 50 students :

Marks	No. of Students	Marks	No. of Students
-------	-----------------	-------	-----------------

More than 0	50	More than 30	20
-------------	----	--------------	----

“ 10	46	“ 40	10
------	----	------	----

“ 20	40	“ 50	3
------	----	------	---

Calculate the median marks.

Q12. Compute median :

Daily Wages (Rs.) : 0-10 10-20 20-30 30-40 40-50 50-60

No. of Workers : 3 9 15 30 18 5

Q13. Calculate median from the following data :

Marks : 10 - 20 20 - 30 30 - 40 40 - 50 50 - 70 70 - 100

Students : 12 20 30 20 12 6

Q14. Calculate median from the following data :

Marks : 0 - 10 10 - 20 20 - 40 40 - 60 60 - 90 90 - 110 110 - 120

Students : 10 12 20 20 22 6 8

Q15. The actual blood pressure values for nine randomly selected individuals are

118.6 127.4 138.4 130 113.7

122 108.3 131.5 133.2

(i) What is the median of the reported blood pressure values?

(ii) Suppose the blood pressure of the second individual is 127.6 rather than 127.4. How does this affect the median of the reported values?

Q16. Consider the sample :

0.736 0.863 0.865 0.913 0.915 0.937

0.983 1.007 1.011 1.064 1.109 1.132

1.140 1.153 1.253 1.394

(i) Compute & compare the values of the sample mean and median.

(ii) By how much could the largest sample observation be decreased without affecting the value of the median?

Q17. The mean and median of the marks of a group of 75 students in a test were calculated as 27 and 29 respectively. It was later discovered that two observations were wrongly recorded as 53 and 45 instead of 35 and 55. Examine to what extent the calculated value of the above two measures will be affected by the discovery of the errors?

Q18. Given below is the pre-tax monthly income of residents of an industrial town:

Pre-tax income (Rs.)	No. of residents (in thousands)
----------------------	---------------------------------

More than 7000	2
----------------	---

More than 6000	8
----------------	---

More than 5000	10
----------------	----

More than 4000	15
----------------	----



More than 3000	25
More than 2000	40
More than 1000	55
More than 0	60

Draw a 'less than' ogive and hence find out :

- (i) the highest income of the lowest 50% of the residents;
- (ii) the minimum income earned by the top 5% of the residents.

### **Trimmed Mean**

Q19. Find the mean and median of the following data :

350 408 540 555 575 590 608 679 815 1285

Also find the 20% trimmed mean by eliminating 2 items from each side. Find 15% trimmed mean.

[Hint : 15% trimmed mean is computed by averaging 10% & 20% trimmed mean]

Q20. Find the mean and median of the following data :

2.0 2.4 2.5 2.6 2.6 2.7 2.7 2.8 3.0 3.1 3.2 3.3 3.3

3.4 3.4 3.6 3.6 3.6 3.6 3.7 4.4 4.6 4.7 4.8 5.3 10.1

Also find the trimmed mean by eliminating 2 items from each side, i.e., 7.7% trimmed mean. Find 11.5% trimmed mean.

Q21. Consider the following sample :

15, 13, 18, 14.5, 12, 11, 8.9, 8

- (i) Find the arithmetic mean, median & 12.5% trimmed mean and compare these values.
- (ii) By how much can the smallest sample observation be increased so that the sample median remains the same?

Q22. Consider the following sample :

27, 6, 24, 30, 50, 2, 150, 44, 80, 36

Find the arithmetic mean and median

- (i) Find the 15% trimmed mean
- (ii) Which of these three measures best summarizes these data? Give reasons.
- (iii) By how much can the smallest sample observation be increased so that the sample median remains the same?
- (iv) By how much can the largest sample observation be increased so that the sample median remains the same?

Q23. The number of calls received per hour at a call centre by the different operators are given below :

5, 4, 7, 6, 8, 10, 11, 0, 7, 18

- (i) Find the arithmetic mean and median.
- (ii) If the last observation is 28 instead of 18, how would the mean and median change?
- (iii) Calculate a 15% trimmed mean.



Q24. The following data consists of the information on the number of cold drinks sold at a store on a particular day.

73, 80, 36, 75, 68, 82, 50, 48, 41, 60.

- Find the sample mean and median.
- If the 3rd observation was 70 instead of 36, how would the sample mean and median change?
- Calculate a 18% trimmed mean.

### MA Entrance

Q25. The frequency distribution of variable X (monthly family expenditure in Rs. '000) for 100 households is as follows : **[DSE MA Ent. Eco. 2008]**

X	10 – 15	5 – 9.9999	2 – 4.9999	Below 2
No. of households	10	15	45	30

The median monthly family expenditure is in the range :

- Rs. 2000 to 3000
- Rs. 3000 to 4000
- Rs. 4000 to 5000
- Rs. 5000 to 6000

### Answers of Exercise 4

- 21,
- 74,
- 133.25,
- 35,
- 20,
- 1,
- $\bar{x} = 128.33$ ,  $M_d = 128.27$ ,
- Median = 51.17, Mode = 52.83,
- 84.1,
- 77.86,
- 27.5,
- 34.3,
- 36,
- 52,
- (i) 127.4, (ii) new median = 127.6,
- (i)  $\bar{X} = 1.03$ , Median = 1.009, (ii) 0.383,
- New Mean = 26.893, New Median = 29,
- (i) Rs. 2,667, (ii) Rs. 6,833,
- $\bar{X} = 640.5$ , Med. = 582.5,  $\bar{X}_{tr(20)} = 591.17$ ,  $\bar{X}_{tr(15)} = 593.71$
- $\bar{X} = 3.65$ , Med. = 3.35,  $\bar{X}_{tr(7.7)} = 3.42$ ,  $\bar{X}_{tr(11.5)} = 3.395$
- (i)  $\bar{X} = 12.55$ , Med. = 12.5,  $\bar{X}_{tr(12.5)} = 12.4$ , (ii) 4,
- $\bar{X} = 44.9$ , Median = 33, (i) 36.15, (ii) trimmed mean, (iii) 28,
- (iv) by any value, 25. (b)

### Basic Concepts

1. Write a short note on Mode.

**Ans.** The value in the series which occurs most frequently or which has the highest frequency is called the mode. But it is not exactly true for every frequency distribution, rather it is the value around which the concentration of the data is very high. According to *Croxtan and Cowden*, "The mode of a distribution is the value at the point around which the items tend to be most heavily concentrated. It may be regarded as the most typical or fashionable value of the series." It is the value which has the greatest frequency density in its immediate neighbourhood. It is



represented by  $M_o$ . It may be noted that a distribution may have one mode, two modes or several modes.

According to Zizek, "The value occurring most frequently in a series (or group) of items and around which the other items are distributed most densely."

If the distribution has only one mode that it is called *Unimodal*, If it has two modes then it is called *Bimodal*, and if it has more than two modes then it is called *Multimodal*.

## 2. Calculation of Mode.

For Individual and Discrete series most frequent value is mode

**Continuous Series :** For continuous series mode is calculated by using the formula given as under :

$$\text{Mode} = L_1 + \frac{|f - f_0|}{|f - f_0| + |f - f_1|} \times i$$

where,  $L_1$  is the lower limit of modal class,

$f$  is the frequency of modal class,

$f_0$  is the frequency of class preceding the modal class,

$f_1$  is the frequency of class succeeding the modal class,

$i$  is the class size.

## 3. Can Mode be determined graphically ?

**Ans.** Yes, Mode can be determined graphically. It can only be determined from a class with highest frequency. Also if the modal class and the class with highest frequency are not adjoining and separated by gap, the mode can be determined graphically. But it has the following exceptions :

- (i) Mode cannot be determined graphically if there are two or more classes having highest frequency.
- (ii) When modal class is adjoining to the class with highest frequency, mode cannot be determined graphically in the modal class.

**Steps required for graphical location of Mode :**

- (i) Prepare a histogram of the given data.
- (ii) The highest rectangle will be the modal class.
- (iii) Draw two lines diagonally in the inside of the modal class rectangle to the upper corner of the adjacent bar.
- (iv) From the point of intersection of these lines, draw a perpendicular of x-axis which gives the modal value.

## 4. Write down the merits of mode.

**Ans.** The merits of mode are given as under :

- (i) The mode is a measure which actually indicates what many people incorrectly believe the arithmetic mean indicates. In certain situations mode



is the only suitable average, e.g., modal size of garments, modal size of shoes, modal wages etc.

- (ii) It is not affected by extreme values.
- (iii) Its value can be determined in open-end distribution without ascertaining the class-limits.
- (iv) It can be used to describe qualitative phenomenon.
- (v) Its value can be determined graphically.
- (vi) It indicates the point of maximum concentration in case of highly skewed or non-normal distributions.
- (vii) It is easy to compute, it can be determined without much mathematical calculations. In individual and discrete series it can be computed even by inspection.
- (viii) Since mode is the most common item of a series it is the best representative value of the series. It is not an isolated example like median. Unlike arithmetic mean, it can not be a value which is not found in the series.
- (ix) Compared to mean, mode is less affected by marginal values in the series. Mode is determined only by the value with highest frequencies.
- (x) The calculation of mode does not require knowledge of all the items and frequencies of a distribution. In simple series, it is enough if one knows the item with highest frequency in the distribution.

**5. Write down the de-merits of mode.**

**Ans.** The de-merits of mode are given as under :

- (i) In case of bi-modal or multi-modal series, mode cannot be determined.
- (ii) It is not capable of algebraic treatment, e.g., combined mode of two or more series cannot be calculated.
- (iii) It is not based on all the items.
- (iv) It is not rigidly defined measure because different formulae gives somewhat different answers.
- (v) Its value is affected significantly by the size of the class-intervals.
- (vi) It is not suitable when relative importance of items is desired.
- (vii) Calculation of mode involves complex procedure of grouping the data.
- (viii) When frequencies of all the items are identical then it is difficult to estimate the mode.

**6. Empirical relationship between Mean, Median and Mode :**

$$\text{Mode} = 3\text{Median} - 2\text{Mean} \quad \text{Or} \quad \text{Median} = \frac{1}{3}(\text{Mode} + 2\text{Mean})$$

**7. Compare the various characteristics of mean, median and mode.**

**Ans.** The following table gives the comparative evaluation of characteristics of mean, median and mode.

S. No.	Characteristics	Measures of Central Tendency
--------	-----------------	------------------------------



		Mean	Median	Mode
1	It is rigidly defined.	Yes	Yes	No
2	It is situated in the centre of the distribution.	No	Yes	Yes
3	It is easily understandable.	Yes	Yes	Yes
4	Its calculation is easy.	Yes	Yes	No
5	It is based on all the observations.	Yes	No	No
6	It is capable of further mathematical treatment.	Yes	No	No
7	It is affected by the choice of sample.	Yes	No	No
8	It is affected by extreme values.	Yes	No	No
9	It can be represented graphically.	No	Yes	Yes

### **Exercise 5**

Q1. Calculate mode from the following data :

Marks	No. of Students	Marks	No. of Students
Below 10	4	Below 60	86
“ 20	6	“ 70	96
“ 30	24	“ 80	99
“ 40	46	“ 90	100
“ 50	67		

Q2. The number of fully-formed tomatoes on 100 plants were counted with the following results :

2 plants had 0 tomatoes	12 plants had 6 tomatoes
5 plants had 1 tomatoes	8 plants had 7 tomatoes
7 plants had 2 tomatoes	6 plants had 6 tomatoes
11 plants had 3 tomatoes	4 plants had 9 tomatoes
18 plants had 4 tomatoes	3 plants had 10 tomatoes
24 plants had 5 tomatoes	

- How many tomatoes were there in all?
- What was the average number of tomatoes per plant?
- What was the modal number of tomatoes ?

Q3. Calculate mean, median and mode from the data given below :

Mid-Value :	5	10	15	20	25	30
Frequency :	5	15	25	30	15	10

Q4. Calculate mode from the following data :

Monthly Wages (in Rs.)	No. of Workers	Monthly Wages (in Rs.)	No. of Workers
200 – 250	4	400 – 450	33
250 – 300	6	450 – 500	17
300 – 350	20	500 – 550	8
350 – 400	12	550 – 600	2



Calculate mode by graphical method also.

- Q5. Find out the mode of the following data graphically and check the result through calculation :

Size	Frequency	Size	Frequency
0 – 1	3	6 – 7	14
1 – 2	7	7 – 8	12
2 – 3	9	8 – 9	8
3 – 4	15	9 – 10	6
4 – 5	25	10 – 11	2
5 – 6	20		

- Q6. From the data given below calculate mode. Also construct a histogram and locate mode.

Value	Frequency	Value	Frequency
0 – 5	328	20 – 25	598
5 – 10	350	25 – 30	524
10 – 15	720	30 – 35	378
15 – 20	664	35 – 40	244

- Q7. Draw a histogram and find mode :

Marks :	10 - 20	20 - 30	30 - 40	40 - 50	50 - 70	70 - 100
Students :	12	20	30	20	12	6

- Q8. Compute mode from the following data relating to dividend paid by joint stock companies in the year 1976 :-

Dividend (in % of the share value)	No. of Companies	Dividend (in % of the share value)	No. of Companies
0 – 1.0	10	10.0 – 12.5	104
1.0 – 2.5	15	12.5 – 14.0	250
2.5 – 3.5	20	14.0 – 15.0	164
3.5 – 4.0	23	15.0 – 17.5	278
4.0 – 5.0	30	17.5 – 19.0	100
5.0 – 7.5	172	19.0 – 20.0	72
7.5 – 9.0	60	20.0 – 22.5	331
9.0 – 10.0	39	22.5 – 25.0	331

- Q9. Find the value of missing frequencies for the following incomplete distribution by appropriate formula when mode value given is 36 :

Variable :	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency :	5	7	-	-	10	6	50

- Q10. Find the missing frequencies in the following table if the mode is 39 and total students are 70.

Marks :	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students :	3	7	-	20	-	10

- Q11. From the following wage distribution find the missing frequencies if Median and Mode are given as Rs. 33.5 and Rs. 34 respectively.



Wages (Rs.):	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequencies:	4	16	?	?	?	6	4	230

- Q12. Determine the values of Median and Mode of the following distribution graphically. Verify the results by actual calculations. After verifying calculate the value of Mean and sketch a curve indicating the general shape of the distribution and comment :

Size	:	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	:	11	19	21	16	10	8	6	3	1

- Q13. The following table shows the earnings of students in their summer jobs

Earnings (Rs.)	Number of Students
0 – 500	12
500 – 1000	19
1000 – 1500	26
1500 – 2000	21
2000 – 2500	13
2500 – 3000	7
3000 – 3500	2

- (i) Calculate the arithmetic mean and mode of the above distribution.  
(ii) Use the result of (i) to comment on the symmetry of the distribution. Give reasons for your answer.

### **Empirical Relationship between Mean, Median and Mode**

- Q14. Define Mean, Median and Mode. Given mean = 70.2 and mode = 70.5. Find median using empirical relationship among them.
- Q15. In a moderately skewed distribution, the arithmetic means 10 units and the mode is 7 units, find the median.
- Q16. In a moderately skewed distribution the value of Arithmetic mean and mode is 20 and 16 respectively. Find out the value of median.
- Q17. In a moderately skewed distribution the mode and mean are 32.1 and 35.4 respectively. Calculate median.
- Q18. In a moderately asymmetrical series the value of mode and median is 20 and 24 respectively. Locate the value of mean.
- Q19. In a moderately asymmetrical series the value of mode and median is 120 and 140 respectively. Find the value of mean.
- Q20. In a moderately asymmetrical series the value of mean is 75 and value of mode is 60. Find the value of median.
- Q21. Find the missing figure:  
Median = mode + ? (Mean- Mode).

### **Answers of Exercise 5**

1. Mean = 42.2, Median = 41.9 and Mode = 41.3, 2.(i) = 100, (ii) 4.86,  
(iii) 5, 3. Mean = 18.25, Median = 18.33 and Mode = 18.75, 4. 428.38,



5. 4.67, 6. 17.3, 7. 35, 8. 14.238, 9. 8 and 14,  
10. 11 & 19, 11. 60, 100 & 40,  
12. Median = 36.6, Mode = 32.35, Mean = 38.725, Distribution is positively skewed  
13.(i) Mean = 1415, Mode = 1291.67, (ii) Positively Skewed,  
14. 70.3, 15. 9, 16. 18.67, 17. 34.3, 18. 26, 19. 150,  
20. 70, 21. 2/3

### Basic Concepts

1. **Partition Values :** Those values which divide the data into  $n$  equal parts are called partition values.
2. **Median :** Median divides the data into two equal parts. Its computation has already been discussed above.
3. **Quartiles :** Quartiles are the partition values which divide the data into four equal parts. We have three quartiles viz.,  $Q_1$  (lower quartile),  $Q_2$  (median) and  $Q_3$  (upper quartile).
4. **Computation of Quartiles :** The quartiles are computed as under :

- (i) **Individual and Discrete series :**

$$Q_1 = \left( \frac{n+1}{4} \right)^{th} \text{ item and } Q_3 = 3 \left( \frac{n+1}{4} \right)^{th} \text{ item}$$

- (ii) **Continuous Series :**

$$Q_1 = l_1 + \frac{\frac{n}{4} - cf}{f} \times i, \text{ and}$$
$$Q_3 = l_1 + \frac{\frac{3n}{4} - cf}{f} \times i$$

where,  $L_1$  is the lower limit of quartile class,

$c.f.$  is the cumulative frequency of class preceeding the quartile class,

$f$  is the frequency of quartile class, and

$i$  is the class size of quartile class.

5. **Deciles :** Deciles are the partition values which divide the data into ten equal parts. We have nine deciles  $D_1, D_2, \dots, D_9$ .
6. **Computation of Deciles :** The deciles are computed as under :

- (i) **Individual and Discrete series :**

$$D_1 = \left( \frac{n+1}{10} \right)^{th} \text{ item, } D_2 = 2 \left( \frac{n+1}{10} \right)^{th} \text{ item, } \dots \dots \text{ and } D_9 = 9 \left( \frac{n+1}{10} \right)^{th} \text{ item}$$

In general the formula is

$$D_r = r \left( \frac{n+1}{10} \right)^{th}$$

- (ii) **Continuous Series :**

$$D_1 = l_1 + \frac{\frac{n}{10} - cf}{f} \times i, D_2 = l_1 + \frac{\frac{2n}{10} - cf}{f} \times i, \dots \dots \text{ and } D_9 = l_1 + \frac{\frac{9n}{10} - cf}{f} \times i,$$

In general the formula is



$$D_r = l_1 + \frac{\frac{r}{10} - cf}{f} \times i$$

where,  $L_1$  is the lower limit of decile class,

$c.f.$  is the cumulative frequency of class preceeding the decile class,

$f$  is the frequency of decile class, and

$i$  is the class size of decile class.

7. **Percentiles** : Percentiles are the partition values which divide the data into hundred equal parts. We have ninety-nine percentiles  $P_1, P_2, \dots, P_{99}$ .
8. **Computation of Percentiles** : The Percentiles are computed as under :

(i) **Individual and Discrete series :**

$$P_1 = \left(\frac{n+1}{100}\right)^{th} \text{ item}, P_2 = 2\left(\frac{n+1}{100}\right)^{th} \text{ item}, \dots \text{and } P_{99} = 99\left(\frac{n+1}{100}\right)^{th} \text{ item}$$

In general the formula is

$$P_r = r\left(\frac{n+1}{100}\right)^{th}$$

(ii) **Continuous Series :**

$$P_1 = l_1 + \frac{\frac{n}{100} - cf}{f} \times i, P_2 = l_1 + \frac{\frac{2n}{100} - cf}{f} \times i, \dots \text{and } P_{99} = l_1 + \frac{\frac{99n}{100} - cf}{f} \times i,$$

In general the formula is

$$P_r = l_1 + \frac{\frac{rn}{100} - cf}{f} \times i$$

where,  $L_1$  is the lower limit of percentile class,

$c.f.$  is the cumulative frequency of class preceeding the percentile class,

$f$  is the frequency of percentile class, and

$i$  is the class size of percentile class.

### **Exercise 6**

#### **Quartiles**

- Q1. Calculate quartiles from the following data  
12, 17, 20, 5, 34, 23, 10, 16, 25, 35, 7
- Q2. Find quartiles from the following data  

X	:	5	10	15	20	25	30	35	40
f	:	2	8	10	14	15	6	25	20
- Q3. Find lower quartile and upper quartile from the following data :  

Wages	:	0-10	10-20	20-30	30-40	40-50
No. of workers	:	22	38	46	35	20
- Q4. Calculate quartiles from the following data :  

Marks	:	0 – 10	10 - 20	20 - 40	40 - 60	60 - 90	90 - 110	110 - 120
Students	:	12	20	30	20	12	6	8



Q5. The rate of sales tax as a percentage of sales, paid by 400 shopkeepers of a market during an assessment year raised from 0 to 25%. The sales tax paid by 18% of them was not greater than 5%. The median rate of sales tax was 10% and 75<sup>th</sup> percentile rate of sales tax was 15%. If only 8% of shopkeepers paid sales tax at a rate greater than 20% but not greater than 25%, summarise the information in the form of frequency distribution taking intervals of 5%. Also find the modal rate of sales tax.

Q6. In 500 small scale industrial units, the return on investment ranged from 0 to 30%, no unit sustaining any loss. 55 of industrial units had returns exceeding 0% but not exceeding 5%. 15% of units had returns exceeding 5% but not exceeding 10%. Median and upper quartile rate of return was 15% and 20% respectively. The uppermost layer of returns exceeding 25% but not exceeding 30% was earned by 25%. Present this information in the form of frequency table with intervals as follows :

Exceeding 0% But not exceeding 5%

Exceeding 5% But not exceeding 10%

Exceeding 10% But not exceeding 15%

Exceeding 15% But not exceeding 20%

Exceeding 20% But not exceeding 25%

Exceeding 25% But not exceeding 30%

Use  $\frac{N}{4}$ ,  $\frac{2N}{4}$ ,  $\frac{3N}{4}$  as ranks of lower, middle and upper quartiles respectively. Find

the rate of return around which there is maximum concentration of units.

Q7. The monthly salary distribution of 250 families in a certain locality in Agra is given below :

Monthly salary	No. of Families
More than 0	250
More than 500	200
More than 1000	120
More than 1500	80
More than 2000	55
More than 2500	30
More than 3000	15
More than 3500	5

find out:

- Limits of the income of middle 50% of the families; and
- If income-tax is to be levied on families, whose income exceeds Rs. 1,800 p.m. calculate the percentage of families, which will be paying income-tax.

Q8. The following table gives the distribution of monthly income of 600 families in a certain city :

Monthly



**Income** : Below 75   75-150   150-225   225-300   300-375   375-450   450 & over  
**No. of**

**Families** :      60            170            200            60            50            40            20

Compute the median income and obtain the limits of income of central 50% of the observed frequencies.

**Deciles**

Q9. Find first and fifth deciles from the following information :

X	:	5	10	15	20	25	30	35	40
f	:	2	8	10	14	15	6	25	20

Q10. From the following distribution of marks in law obtained by 50 students :

Marks	No. of Students	Marks	No. of Students
More than 0	50	More than 30	20
“      10	46	“      40	10
“      20	40	“      50	3

Calculate the second and eighth deciles marks.

**Percentiles**

Q11. Define a Percentile. Find the 45<sup>th</sup> and 57<sup>th</sup> percentiles for the following data on marks obtained by 100 students :

Marks	:	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	:	10	20	20	15	15	20

Q12. If 25% of the observations lie above 80, 40% of the observations are less than 50 and 70% are greater than 40, then ... = 80; ... = 50 and ... = 40.

Q13. The following are the marks obtained by 30 students in Statistics :

Marks	No. of Students
10 Marks or less	4
20 Marks or less	10
30 Marks or less	30
40 Marks or less	40
50 Marks or less	47
60 Marks or less	50

Draw a curve on the graph paper and show therein :

- (i) The range of marks by middle 80% of the students.
- (ii) The Median.

Q14. With the help of given data, find :

- (i) Value of middle 50% item ;
- (ii) Value of exactly 50% item ;
- (iii) The value of  $P_{40}$  and  $D_6$  ;
- (iv) Graphically with the help of ogive curve, the values of  $Q_1$ ,  $Q_3$ , median,  $P_{40}$  and  $D_6$  ;



Class

Interval	:	10–14	15–19	20–24	25–29	30–34	35–39	Total
Frequencies	:	5	10	15	20	10	5	65

Q15. The marks obtained by 70 students in an examination are grouped as follows :

Marks	:	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of Students	:	4	8	10	25	7	16

Calculate :

- The marks at the 25<sup>th</sup> percentile.
- The marks at the third quartile.
- The approximate number of students who failed in the examination if the pass marks are considered to be 40%.

Q16. From the following data, calculate the percentage of workers getting wages  
(a) more than Rs. 44, (b) between Rs. 22 and Rs. 58, (c) the quartile deviation  
wages : 0-10 10-20 20-30 30-40 40-50 50-60 60-70 70-80

No. of workers:	20	45	85	160	70	55	35	30
-----------------	----	----	----	-----	----	----	----	----

**Answers of Exercise 6**

- $Q_1 = 10, Q_3 = 23,$
- $Q_1 = 20, Q_3 = 35,$
- $Q_1 = 14.803, Q_3 = 34.21,$
- $Q_1 = 17.5, Q_3 = 59,$
- CI: 0 – 5 5 – 10 10 – 15 15 – 20 20 – 25  
 $f : 72 \quad 128 \quad 100 \quad 68 \quad 32$  Mode = 8.33,
- $a = 120, b = 125, c = 0,$
- $Q_1 = 578, Q_3 = 1850, 26%,$
- Med = 176.2,  $Q_1 = 114.7, Q_3 = 250,$
- $D_1 = 15, D_5 = 30,$
- $D_2 = 20, D_8 = 40,$
- $P_{45} = 33.75, P_{57} = 37.33,$
- Med = 176.2,  $Q_1 = 114.7, Q_3 = 250,$
- $P_{75}$  or  $Q_3 = 80, P_{40} = 50, P_{30} = 40,$
- 14.(i)  $Q_1 = 19.92, Q_3 = 29.19,$  (ii) Med. = 25.75, (iii)  $P_{40} = 23.17, D_6 = 29,$
- 15.(i)  $P_{25} = 25.5,$  (ii)  $Q_3 = 47.86,$  (iii) 16.
- 16.(a) 32.4%, (b) 68.4%, (c) 11.115,

For Full Course Video Lectures of  
ALL Subjects of Eco (Hons), B Com (H), BBE & MA Economics  
Register yourself at  
[www.primeacademy.in](http://www.primeacademy.in)

**Dheeraj Suri Classes**

**Prime Academy**

+91 9899192027

**Statistical Methods 2.37**

**For MA Economics Entrance**

By : **Dheeraj Suri, 9899-192027**

**[www.primeacademy.in](http://www.primeacademy.in)**