Chapter 10

Linear Programming

Learning Objectives :

After learning this chapter you will understand :

- **Linear Programming Problem.**
- **Formation of Linear Programming Problem.**
- **Solution of a LPP**
  - ✓ **Solution of LPP by Graphical Method.**
  - ✓ **Solution of LPP by Simplex Method.**
- **Unbounded Solution.**
- **Multiple Optimal Solutions.**
- **Degeneracy.**
- **Solution of a LPP by Big M Method.**
- **Solution of a Minimisation Problem by Simplex Method.**
- **Duality Theory.**

Basic Concepts

1. **Inequation :** A mathematical statement comparing two terms with less than (<) or greater than (>) sign is known as inequation.
2. **Linear Programming :** Linear programming is an optimization technique for finding an optimal solution (maximum or minimum) for an objective function, which is a linear function of two or more variables, subject to various constraints in the form of linear inequations (or equations).

> *Note :* Every linear programming problem has two important features :
> (i) an objective function to be maximized or minimized.
> (ii) constraints or restrictions.

3. **Objective function :** A linear function of two or more variables which has to be maximized or minimized, under the given restrictions in the form of linear inequations, is called objective function.
4. **Linear constraints :** The linear inequations in one or more variables are called the linear constraints.
5. **Non-negativity constraint :** A set of constraints that require the value of each decision variable to be either zero or more than zero.
6. **Solution of a Linear Programming Problem :** A set of values of decision variables which satisfy all the linear constraints of a linear programming problem is called the solution of such LPP.
7. **Feasible solution :** Any solution of a LPP which also satisfies the non negativity restrictions of such LPP is called feasible solution.
8. **Feasible region :** The set of all feasible solutions is called the feasible region.
9. **Infeasible solution :** Any point lying outside the feasible region of the LPP. It violates any one or more constraints.
10. **Optimal feasible solution :** Any point in the feasible region which maximizes or minimizes the objective function is called the optimal solution.
11. **Optimization technique :** The processes of obtaining the optimal solution are called optimization technique.
12. **General model of LPP :** The general model of a LPP having n decision variables and m constraints can stated in the following form :
Maximize or minimize $Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots.. + c_nx_n$
Subject to the linear constraints :
$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots.. + a_{1n}x_n (\leq, =, \geq) b_1$
$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots.. + a_{2n}x_n (\leq, =, \geq) b_2$
....
....
$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots.. + a_{mn}x_n (\leq, =, \geq) b_m$
$and,\ x_1, x_2, \ldots\ldots\ldots. x_n \geq 0$
where,
(i) Z is the objective function

(ii) $x_1, x_2, x_3, ...x_n$ are decision variables.

(iii) In the set of constraints $(\leq, =, \geq)$ means that each constraint may have any one of these three sign.

13. **Steps required to Form a LPP :** To form a LPP from the given problem the following steps are taken :

(i) Identify decision variables from the given problem and denote them by x_1, x_2, …..x_n.

(ii) Form the objective function expressing it as a linear combination of decision variables.

(iii) Identify the constraints and express them in the form of linear inequations (or equations) of decision variables.

(iv) Add the non-negativity restriction on the decision variables, because the physical objects can never have negative values.

Exercise 1

Formation of a Linear Programming Problem :

Q1. Two tailors, A and B, earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimize the labour cost to produce at least 60 shirts and 32 pants.

Q2. A factory owner has to purchase two types of machines, A and B, for his factory. The requirements and limitation for the machines are as follows :

	Area occupied by the machine	Skilled men for each machine	Daily output in units
Machine A	1000 sq. M.	12 men	60
Machine B	1200 sq. M.	8 men	40

He has an area of 9000 sq m. available and 72 skilled men who can operate the machines. Form a LPP to determine how many machines of each type should he buy to maximize the daily output?

Q3. A company uses three machines to manufacture two types of shirts, of half sleeves and full sleeves which they make and sell. The number of hours required on machines M_1, M_2, M_3, for one shirt of each type are as follows:

	M_1	M_2	M_3
Half sleeves	1	2	8/5
Full sleeves	2	1	8/5

None of the machines can be operated for more than 40 hours per week. The profit on each half sleeves shirt is Rs. 1 and profit on each full sleeves shirt is Rs. 1.50. Form the above problem as LP model to find how many of each type of shirts should be made in maximizing the company's profits? DO NOT SOLVE

Q4. Shyam, an agriculturist, has a farm with 125 acres. He produces radish, peas and potato. Whatever he raises, is fully sold in the market. He gets Rs. 5 for radish per kg., Rs. 4 for peas per kg and Rs. 5 for potato per kg. The average yield is 1500 kg of radish per acre, 1800 kg of peas per acre and 1200 kg of potato per acre. To produce each 100 kg of radish and peas and to produce each 80 kg of potato a sum of Rs.

12.50 has to be used for manure. Labour required for each acre to raise the crop is 6 man-days for radish and potato each and 5 days for peas. A total of 500 man-days of labour at rate of Rs. 40 per man-days are available. Formulate this as a linear programming model to maximize the Agriculturist's total profit. **[B. Com (H) 2001]**

Q5. A person is interested in investing Rs. 50,00,000 in a mix of investments. The investment choices and expected rates of return on each one of them are:

Investment	***Projected Rate of return***
Mutual Fund A	0.12
Mutual Fund B	0.09
Money Market Fund	0.08
Government bonds	0.085
Share Y	0.16
Share X	0.18

The investor wants at least 35 percent of his investment in Government bonds. Because of the higher perceived risk of two shares, he has specified that the combined investment in these not to exceed Rs. 80,000. The investor also has specified that at least 20 percent of the investment should be in the money market fund and that the amount of money invested in shares should not exceed the amount invested in mutual funds. His final investment conditions is that the amount invested in mutual fund A should be not more than the amount invested in mutual fund B. The problem is to decide the amount of money to invest in each alternative so as to obtain the highest annual total return. Formulate the above as a linear programming problem. **[B. Com (H) 1994]**

Q6. A Mutual Fund Co. has Rs. 15 lakhs available for investment in government bonds, blue-chip stocks, speculative stocks and short term bank deposits. The annual expected returns and risk factors are given below :

Investment Type	**Expected Return**	**Expected Risk** (0 to 100)
Government Bonds	9%	11
Blue-chip Stock	20%	25
Speculative Stocks	35%	45
Short-term Deposits	6%	5

The mutual fund is required to keep at least Rs. 2 lakhs in short term deposits and not to exceed an average risk factor of 45. Speculative bonds must be at the most 15 % of the total amount invested. How should the mutual fund invest to maximize its total expected annual return. Formulate this as a linear programming problem. DO NOT SOLVE. **[B. Com. (H) 2007(C)]**

Q7. A 24 hour supermarket has the following minimal requirements for cashiers:

Period :	1	2	3	4	5	6
Time of day (24 hour clock) :	3 - 7	7 - 11	11 - 15	15 - 19	19 - 23	23 - 3
Minimum no. required :	7	20	14	20	10	5

Period 1 follows immediately after period 6. A cashier works eight consecutive hours, starting at the beginning of one of the six periods. To determine a daily employee work sheet which satisfies the requirements with the least number of personnel formulate the problem as a linear programming problem. **[B. Com (H) 2000]**

Q8. A city hospital has the following minimum requirements for nurses:

Clock Time (24 hours per day)	*Minimum number of Nurses required*
6 A.M. – 10 A.M.	2
10A.M. – 2 P.M.	7
2 P.M. – 6 P.M.	15
6 P.M. – 10 P.M.	8
10 P.M. – 2 A.M.	20
2 A.M. – 6 A.M.	6

Nurses report to the hospital at the beginning of each period and work for 8 consecutive hours. The hospital wants to determine the minimum number of nurses to be employed so that there will be sufficient number of nurses available for each period. Formulate this as a linear Programming Problem by setting up appropriate constraint, and objective function. Do not solve. **[B. Com (H) 2006(R)]**

Q9. A drug manufacturer desires to obtain a production schedule to maximize the net revenue from the sale of two drugs A and B whose net revenues per unit are Rs. 60 and Rs. 40 respectively. The constraints are given below :

(i) capacity to produce drugs is 1000 units of A and B together,

(ii) capacity to produce containers is 1600 units of A or 800 units of B,

(iii) labour is available for 800 units of A and 1600 units of B.

What product mix of A and B should be produced so as to maximize net revenue ? Formulate the problem as LP model and DO NOT SOLVE.

Q10. A manufacturer produces two different models X and Y of same product. The raw materials A and B are required for production. At least 18 kg of A and 12 kg of B must be used. Also at most 34 hours of labour are to be utilized. 2 kg of A are needed for each model X and 1 kg of A, for each model Y. For each model X and Y, 1 kg of B is required. It takes 2 hours to manufacture a model X and 1 hour to manufacture a model Y. The profit is Rs. 50 for each model X and Rs. 30 for each model Y. Formulate the above as LPP to determine how many units of each model should be produced to maximize profit. Do Not Solve.

Answers of Exercise 1

1. Min. $Z = 150x_1 + 200x_2$, Subject to constraints
$6x_1 + 10x_2 \geq 60$, $4x_1 + 4x_2 \geq 32$, $x_1, x_2 \geq 0$,

2. Max. $Z = 60x_1 + 40x_2$, Subject to constraints
$1000x_1 + 1200x_2 \leq 9000$, $12x_1 + 8x_2 \leq 72$, $x_1, x_2 \geq 0$,

3. Max. $Z = x_1 + 1.5x_2$, Subject to constraints
$x_1 + 2x_2 \leq 40$, $2x_1 + x_2 \leq 40$, $\frac{8}{5}x_1 + \frac{8}{5}x_2 \leq 40$, $x_1, x_2 \geq 0$,

4. Max. $Z = 7072.5x_1 + 6775x_2, + 5572.5x_3$, Subject to constraints
$x_1 + x_2 + x_3 \leq 125$, $6x_1 + 5x_2 + 6x_3 \leq 500$, $x_1, x_2, x_3 \geq 0$,

5. Max. $Z = 0.12x_1 + 0.09x_2 + 0.08x_3 + 0.085x_4 + 0.16x_5 + 0.18x_6$, Subject to constraints

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 50,00,000$, $x_4 \geq 17,50,000$, $x_5 + x_6 \leq 80,000$,

$x_3 \geq 10,00,000$, $x_5 + x_6 \leq x_1 + x_2$, $x_1 \leq x_2$, $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$,

6. Max. $Z = 0.09x_1 + 0.2x_2 + 0.35x_3 + 0.06x_4$,

Subject to constraints

$x_1 + x_2 + x_3 + x_4 \leq 15,00,000$, $x_4 \geq 2,00,000$, $x_3 \leq 2,25,000$, $x_1, x_2, x_3, x_4, \geq 0$

7. Min. $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$,

Subject to constraints

$x_1 + x_2 \geq 20$, $x_2 + x_3 \geq 14$, $x_3 + x_4 \geq 20$, $x_4 + x_5 \geq 10$,

$x_5 + x_6 \geq 5$, $x_1 + x_6 \geq 7$, $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$,

8. Min. $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$,

Subject to constraints

$x_1 + x_2 \geq 7$, $x_2 + x_3 \geq 15$, $x_3 + x_4 \geq 8$, $x_4 + x_5 \geq 20$,

$x_5 + x_6 \geq 6$, $x_1 + x_6 \geq 2$, $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$,

9. Max. $Z = 60x_1 + 40x_2$,

$x_1 + x_2 \leq 1000$, $x_1/1600 + x_2/800 \leq 1$, $x_1 \leq 800$, $x_2 \leq 1600$, $x_1, x_2 \geq 0$,

Basic Concepts

1. **Graphical method to solve a LPP :** To solve a linear programming problem having at most two variables we use graphical method. It involves the following two techniques :
   (i) Corner point method, and
   (ii) Iso profit or Iso cost method.

> *Note :* Graphical method is not suitable for LPP having more than two variables. They are solved by using simplex method.

2. **Steps required to solve a linear programming problem by corner point method :**
   (i) Write down the equations representing the given inequations.
   (ii) Plot the equations obtained in step (i) on the same graph.
   (iii) Determine the feasible region using the given inequations.
   (iv) Find the coordinates of the corner points of the feasible region obtained in step (iii).
   (v) Evaluate the objective function at each of the corner point obtained in step (iv) to determine its maximum or minimum value.

> *Note :* Corner point method is suitable for LPP having only a few constraints. The problems with more constraints can be handled easily by iso profit (or cost) method.

3. **Unbounded Solution :** When the feasible region of the given LPP is not bounded and the objective function Z may take arbitrarily large values then such type of problems are called problems having unbounded solution.
4. **Multiple Optimal Solution :** If the objective function Z of a LPP has optimal(maximum or minimum) solutions at more than one points then it is said to have multiple optimal solutions.
5. **Infeasibility :** If there is no common (feasible) region between the constraints then such problem is known to have infeasible solution e.g. $x + y \leq 1$; $2x + 5y \geq 10$.

Exercise 2

Q1. Maximize $Z = 10x + 6y$
Subject to
$3x + y \leq 12$; $2x + 5y \leq 34$; $x, y \geq 0$
By graphic method. **[B. Com (H) 1984]**

Q2. Solve the following LPP by graphic method : **[B. Com (H) 2013(C)]**
Maximize $Z = 2x + 5y$
Subject to
$x + 4y \leq 24$; $3x + y \leq 21$; $x + y \leq 9$; $x, y \geq 0$.

Q3. Solve graphically : **[B. Com (H) 2009]**
Max. $Z = 50x + 30y$
Subject to
$2x + y \geq 18$; $x + y \geq 12$; $3x + y \leq 34$; $x, y \geq 0$.

Q4. Find Graphically the minimum of : **[B. Com. (H) 2008]**
$Z = 16x + 25y$
Subject to constraints,
$2x + y \geq 7$, $x + y \geq 5$, $2x + 5y \geq 16$, $x, y \geq 0$

Q5. Solve the following linear programming problem graphically :
Maximize $Z = 3x + 4y$ **[B. Com. (H) III Sem. 2012]**
Subject to
$10x + 8y \leq 80$; $x \geq 2y - 10$; $y \leq 6$; $x, y \geq 0$

Q6. Solve graphically : **[B. Com. (H) 2011]**
Minimize : $z = 200x_1 + 400x_2$
Subject to the constraints :
$x_1 + x_2 \geq 200$, $\frac{1}{4}x_1 + \frac{3}{4}x_2 \geq 100$, $\frac{1}{10}x_1 + \frac{1}{5}x_2 \leq 35$, $x_1, x_2, \geq 0$.

Q7. Solve the following L.P.P. by graphic method :
Minimize $Z = -x_1 + 2x_2$
Subject to constraints,
$-x_1 + 3x_2 \leq 10$, $x_1 + x_2 \leq 6$, $x_1 - x_2 \leq 2$, $x_1, x_2 \geq 0$

Q8. Solve the following L.P.P by graphic method :
Maximize $Z = 2.75x_1+4.15x_2$
Subject to : $2x_1 + 2.5x_2 \leq 100$, $4x_1 + 8x_2 \geq 160$
$7.5x_2 + 5x \geq 150$, $x_1, x_2 \geq 0$. **[B. Com (H) 2001]**

Q9. Minimize Z = x – 5y + 20, subject to constraints
$x - y \geq 0$, $-x + 2y \geq 2$, $x \geq 3$, $y \leq 4$, $x, y \geq 0$

Q10. Solve the following linear programming problem graphically:
Maximise $z = 4x + 6y$ subject to constraints
$x + y = 5$, $x \geq 2$, $y \leq 4$, $x, y \geq 0$. **[B. Com (H) 1992]**

Q11. Using graphic method, find the maximum value of : $z = 7x + 10y$
Subject to $x + y \leq 30{,}000$, $y \leq 12{,}000$
$x \geq 6{,}000$ $x \geq y$ $x, y \geq 0$ **[B. Com (H) 1990, 1996]**

Q12. In linear programming what is a 'Feasible Solution' and 'Infeasible Solution'. Solve the following by graphic method and comment on the result :
Min. : $Z = 3x_1 + 2x_2$
Subject to
$-2x_1 + 3x_2 \leq 9$, $3x_1 - 2x_2 \leq -20$, $x_1, x_2 \geq 0$ **[B. Com. (H) 2007(R)]**

Q13. What are "redundant constraints"? Which constraints are redundant in the following with **[B. Com. (H) 2007(R)]**
$x_1, x_2 \geq 0$ $4x_1 + 3x_2 \leq 12$, $-x_1 + x_2 \geq 1$, $x_1 + x_2 \leq 4$, $x_1 + x_2 \leq 6$.

Q14. In linear programming what is "Multiple Solution Situation". Solve the following graphically and interpret the result : **[B. Com. (H) 2007(C)]**
Max. : $Z = 4x + 3y$
Subject to
$3x + 4y \leq 24$, $8x + 6y \leq 48$, $x \leq 5$, $y \leq 6$, $x, y \geq 0$

Q15. A furniture dealer deals in only two items: table and chair. He has Rs. 5,000 to invest and a space at the most 60 pieces. A table costs him Rs. 250 and a chair Rs. 50. he can sell a table at a profit of Rs. 50 and a chair at a profit of Rs. 15. Assuming he can sell all the items that he buys, how should he invest his money in order that he may maximize his profit? Use Graphical method. **[B. Com (H) 2006(R)]**

Q16. A carpenter has started a workshop in which he manufactures handcarts. Each cart consists of a frame and two wheels. A frame uses 3 hours and a wheel uses 2 hours of labour of which 90 hours per week are available. The carpenter wants that he should manufacture at least 10 carts during a week. The cost of a frame is Rs. 500 and that of wheel is Rs. 200. Formulate the above as a linear programming problem and find its optimum solution. **[B. Com (H) 2004]**

Q17. A carpenter has 90, 80 and 50 running feet respectively of teak, plywood and rosewood. The product A requires 2, 1 and 1 running feet and product B requires 1, 2 and 1 running feet of teak, plywood and rosewood respectively. If A would sell for Rs. 48 and B would sell for Rs. 40 per unit, how much of each should he make and sell in order to obtain the maximum gross income out of his stock of wood.
(i) Give a mathematical formulation to this linear programming problem
(ii) Use graphical method to solve the problem.
(iii) Indicate clearly the feasible region on a graph paper. **[B. Com (H) 2006(C)]**

Q18. A retired person wants to invest up to an amount of Rs. 30,000 in the fixed income securities. His broker recommends investing in two bonds- bond A yielding 7% per annum and bond B yielding 10% per annum. After some consideration he decides to invest at the most Rs. 12,000 in bond B and at least Rs. 6,000 in bond A. He also wants that the amount invested in bond A must be at least equal to the amount invests in bond B. What should the broker recommend if the investor wants to maximize his return on investment? Solve graphically. **[B. Com (H) 1999]**

Q19. Alpha Heavy Engineering Company produces earthmovers and harvesters. Each product passes through two assembly departments A and B, which respectively, have 300 hours and 320 hours of available time for the next month's production. Each earthmover requires 20 hours in department A and 40 in department B and each harvester requires 30 hours in department A and 20 in department B. the two products are tested in a third department. Each earthmovers is given 60 hours of testing and each harvester 20, and , as per the agreement with the labour union, the total labour hours devoted to testing cannot fall below 270. The management has the oprating policy of manufacturing at least one harvester for every two earthworms produced. A major customer has placed an order for a minimum of 5 earthmovers and harvester (in any combination, whatever) for next month, and so, at least that many must be produced. Each earthmover gives a profit of Rs. 10,000 and each harvester Rs. 8,000. Formulate the above as a linear programming problem and solve the same graphically. **[B. Com (H) 2002]**

Q20. A firm assembles and sells two different types of outboard motors A and B using four resources. The production process can be described as follows:

	Resources		*Capacity per month*
1.	Motor Unit Shop	:	400 type A or 250 type B units or any linear combination of the two
II.	Type A gear and drive Shop resource	:	175 type A units
III.	Type B gear and drive Shop resource	:	225 type B units
IV.	Final Assembly Resource	:	200 type A units or 350 type B unit or any linear combination of the two

Type A units bring in a profit of Rs. 90 each and Type B units Rs. 60 each. Formulate the above as a linear programming problem to maximize profit and solve the same by graphic method. Also write its dual and determine the shadow price of the resources. **[B. Com. (H) 2003, 2009]**

Q21. The standard weight of a special purpose brick is 5 kg and it contains two basic ingredients G-I and G-II. G-I costs Rs. 5 per kg and G-II costs Rs. 8 per kg. Strength conditions dictate that brick contains not more than 4 kg of G-I and minimum of 2 kg of G-II. Since the demand for the product is likely to be related to the price of the brick, find the minimum cost of brick satisfying the above condition. Use graphic method. **[B. Com. (H) 2007(C), 2008]**

Q22. A manufacturer produces two different models X and Y of same product. The raw materials A and B are required for production. At least 18 kg of A and 12 kg of B must be used. Also at most 34 hours of labour are to be utilized. 2 kg of A are needed for each model X and 1 kg of A, for each model Y. For each model X and Y, 1 kg of B is required. It takes 2 hours to manufacture a model X and 1 hour to manufacture a model Y. The profit is Rs. 50 for each model X and Rs. 30 for each model Y. Show graphically how many units of each model should be produced to maximize profit. **[B.B.E. 2004]**

Q23. A firm plans to purchase atleast 200 quintals of scrap containing high quality metal X and low quality metal Y. It decides that the scrap to be purchased must contain atleast 100 quintals of X-metal and not more than 35 quintals of Y-metal. The firm can purchase the scrap from two suppliers A and B in unlimited quantities. The percentage of X and Y metals in terms of weight in the scraps supplied by A and B is :

Metals	Supplier A	Supplier B
X	25%	75%
Y	10%	20%

Price of A's scrap is Rs. 200 per quintal and that of B's is Rs. 400 per quintal. Formulate this Linear Programming Problem and solve it graphically to determine the quantities that the firm should buy from the two suppliers so as to minimize total purchase cost. **[B.B.E. 2005]**

Q24. Explain how you would identify the cases of redundant constraints, no solution, multiple solution and unbounded solution from the graph of linear programming problems involving two variables. Give a rough sketch of each case. **[B. Com. (H) 1991]**

Q25. What do you understand by (i) infeasibility and (ii) unbounded solution? How would you identify each one of these in graphic solution to linear programming problems? Draw a rough sketch for each one. **[B. Com. (H) 1995]**

Q26. In the context of linear programming, what do you mean by no optimal solution, multiple optimum solution and unbounded optimum solution? How would you recognize these in the optimum simplex tableau? **[B. Com. (H) 2002]**

Q27. In case of Linear Programming Problems, what is a feasible solution and an infeasible solution? Describe each giving an imaginary graph. **[B. Com. (H) 2003]**

Q28. Show graphically a situation when a LP problem:

(i) has no solution; (ii) has an unbounded solution;

(iii) has multiple solution. **[B. Com. (H) 2005(R)]**

Q29. What do you understand by (i) infeasibility and (ii) unbounded solution? How would you identify each one of these in graphic solution to linear programming problems? Draw a rough sketch for each one. **[B. Com. (H) 2012]**

Answers of Exercise 2

1. $x = 2$, $y = 6$, max. Z = 56, **2.** $x = 4$, $y = 5$ and max. Z = 33,

3. $x = 10$, $y = 2$, Max. Z = 560, **4.** Min. Z = 94.25 at $x = 19/8$, $y = 9/4$,

5. $x = 3.2$, $y = 6$ and max. Z = 33.6, **6.** Min. Z = 60,000 at $x_1 = 100$, $x_2 = 100$,

7. $x_1 = -2$, $x_2 = 0$ & min. value of z is –2, **8.** $x_1=0$, $x_2 =40$ & max. value of z is 166,

9. $x = 4$, $y = 4$ & min. value of z is 4, **10.** x = 2, y = 3 and max. value of z is 26,

11. $x = 18{,}000$, $y = 12{,}000$ and max. value of z is 2,46,000,

12. Infeasible Solution, **13.** $x_1 + x_2 \leq 6$,

14. Max Z = 24 at $x = 3$, $y = 4/3$ and at $x = 24/7$, $y = 24/7$,

15. 50 chairs and 10 tables are produced for a maximum profit of Rs. 1250,

16. 10 frames and 20 wheels are produced for a minimum cost is Rs. 9000,

17. 40 units of product A and 10 units of product B are produced for a maximum income of Rs. 2320,

18. Rs. 18,000are invested in type A and Rs. 12,000 in type B for maximum return of Rs. 2,460,

19. 9/2 earthmovers and 7 harvesters are produced for a maximum profit of Rs. 1,01,000,

20. 800/9 motors of type A and 1750/9 motors of type B. Maximum profit Rs. 19,666.67

21. Min. Z = $5x + 8y$ Subject to $x + y = 5$, $x \le 4$, $y \ge 2$, $x, y \ge 0$ and solution is min z = 31 at $x = 3$, $y = 2$,

22. Max Z = $50x + 30y$, Subject to $2x + y \ge 18$, $x + y \ge 12$, $2x + y \le 34$, $x, y \ge 0$ and solution is $x = 0$, $y = 34$ and max Z = 1020,

23. Min. Z = $200x + 400y$, sunject to $x + y \ge 200$, $0.25x + 0.75y \ge 100$, $0.10x + 0.20y \le 35$, $x, y \ge 0$ and solution is $x = 100$, $y = 100$ and min Z = 60,000

Basic Concepts

Solution of a LPP by Simplex Method :

1. **Slack variable :** It is a variable added to the left hand side of a 'less than or equal to' inequality to convert it into an equality.
2. **Basic feasible Solution :** A basic feasible solution to a linear programming problem is a basic solution for which m variables solved for, are all greater than or equal to zero.
3. **Steps required to solve a L.P.P. by simplex method :**

Let the given problem is

Maximize Z = $c_1x_1 + c_2x_2 + c_3x_3 + \ldots\ldots\ldots\ldots c_nx_n$

Subject to the constraints

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots\ldots\ldots\ldots a_{1n}x_n \le b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots\ldots\ldots\ldots a_{2n}x_n \le b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots\ldots\ldots\ldots a_{3n}x_n \le b_3$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots\ldots\ldots\ldots a_{mn}x_n \le b_m$

$x_1, x_2, \ldots\ldots x_n \ge 0$

Step I : Convert the constraints to equations by adding slack variables $s_1, s_2, \ldots\ldots s_m$ to the left hand side of given inequalities. So, the given problem becomes

Maximize Z = $c_1x_1 + c_2x_2 + c_3x_3 + \ldots\ldots\ldots\ldots c_nx_n + 0s_1 + 0s_2 + 0s_3 + \ldots\ldots + 0s_m$

Subject to the constraints

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots\ldots\ldots\ldots a_{1n}x_n + s_1 = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots\ldots\ldots\ldots a_{2n}x_n + s_2 = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots\ldots\ldots\ldots a_{3n}x_n + s_3 = b_3$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots\ldots\ldots\ldots a_{mn}x_n + s_m =\le b_m$

$x_1, x_2, \ldots\ldots x_n, s_1, s_2, \ldots\ldots s_m \ge 0$

Step II : Set up the initial tableau, the specimen is given as under

$C_j \rightarrow$ $\downarrow$			c_1 c_2 c_3 … c_n 0 0 0 ….. 0	
	Basic Variable	Solution	x_1 x_2 x_3 … x_n s_1 s_2 s_3 …. s_m	Minimum Ratio
0	s_1	b_1	a_{11} a_{12} a_{13} …a_{1n} 1 0 0 ….. 0	
0	s_2	b_2	a_{21} a_{22} a_{23} …a_{2n} 0 1 0 ….. 0	
	⋮	⋮	⋮	
0	s_m	b_m	a_{m1} a_{m2} a_{m3} …a_{mn} 0 0 0 ….. 1	
	Z_j	0	0 0 0 0 0 0 0 0	
	$C_j - Z_j$		c_1 c_2 c_3 … c_n 0 0 0 ….. 0	

Step III : Check the optimality, if all the entries in the $C_j - Z_j$ row are either negative or zero then the solution is optimal. Else proceed to next step.

Step IV : Find the entering variable by determining the pivot (key) column. A pivot column is one having highest positive entry in $C_j - Z_j$ row.

Step V : Find the departing variable by determining the pivot (key) row. A pivot row is the one having the minimum ratio.

Step VI : Find the new simplex tableau and go back to step III.

Exercise 3

Q1. Solve by simplex method :
Maximize $Z = 45x_1 + 80x_2$
Subject to the constraints :
$5x_1 + 20x_2 \leq 400$, $10x_1 + 15x_2 \leq 450$, $x_1, x_2 \geq 0$

Q2. Solve by simplex method :
Maximize $Z = 3x_1 + 4x_2$
Subject to the constraints :
$2x_1 + 2x_2 \leq 80$, $2x_1 + 4x_2 \leq 120$, $x_1, x_2 \geq 0$

Q3. Solve by simplex method :
Maximize $Z = 80x_1 + 60x_2 + 30x_3$
Subject to the constraints :
$10x_1 + 4x_2 + 5x_3 \leq 2000$, $2x_1 + 5x_2 + 4x_3 \leq 1009$, $x_1, x_2, x_3 \geq 0$

Q4. Solve by simplex method :
Maximize $Z = 2x_1 + 4x_2 + x_3 + x_4$
Subject to the constraints :
$x_1 + 3x_2 + x_4 \leq 4$, $2x_1 + x_2 \leq 3$, $x_2 + x_3 + x_4 \leq 3$, $x_1, x_2, x_3, x_4 \geq 0$

Q5. Solve by simplex method :
Maximize $Z = 4x_1 + x_2 + 3x_3 + 5x_4$
Subject to the constraints :
$4x_1 - 6x_2 - 5x_3 - x_4 \geq -20$, $-3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$,
$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$, $x_1, x_2, x_3, x_4 \geq 0$

Q6. Given the following simplex tableau : **[B. Com. (H) 2013(C)]**

$C_j \rightarrow$ $\downarrow$	Basic Variables	X_1	X_2	S_1	S_2	S_3	Quantity
		1	0	1	0	0	4
		0	1	0	1	0	6
		3	0	0	-2	1	6
	Z_j	0	5	0	5	0	
	$C_j - Z_j$	3	0	0	-5	0	

(i) What variables form the basic solution?

(ii) What are the values of $C_1, C_2, \ldots, C_j$?

(iii) Is this the optimal solution? If yes, explain; if not find the optimal solution.

Q7. A firm makes two products, chairs (x) and tables (y), which must be processed on two machines, A and B. The following information is available:

Product	Hours required per unit of Product on machine A	B	Profit per unit (Rs.)
Chair	2	4	6
Table	4	2	8

The maximum hours available on A and B are 60 and 48 respectively.

The optimum simplex tableau for maximizing the profit of the above problem is as under :

$C \rightarrow$ $\downarrow$	Product Mix	Quantity	6 x	8 y	0 s_1	0 s_2
8	y	12	0	1	1/3	-1/6
6	x	6	1	0	-1/6	1/3
	z_j	132	6	8	5/3	2/3
	$c_j - z_j$		0	0	-5/3	-2/3

(s_1 and s_2 are the slack variables of machine A and machine B respectively.)

You are required to explain, in the context of the given problem, the meaning and justification of all the elements in the above simplex tableau. **[B. Com (H) 1991]**

Q8. Given the Simplex tableau for a maximization problem of linear programming:

C_j	*Product Mix*	x_1	x_2	s_1	s_2	*Quantity*
4	x_2	1	1	1	0	6
0	s_2	1	0	-1	1	2
	C_j	3	4	0	0	

(i) Find whether the above table gives optimum solution. If not, improve it and find the optimum solution.

(ii) If s_1 and s_2 are slack variables at machine I and machine II, what is the maximum price you will like to pay for one hour of each machine?

[B. Com. (H) 1997, 2006(R)]

Q9. The XYZ company manufacturers two products A and B. These are processed on same machine. A takes 10 minutes per item and B takes 2 minutes per item on machine. Machine can run for a maximum of 35 hours in a week. Product A requires 1 kg and product B 0.5 kg of the raw material per item, the supply of which is 600 kg per week. Not more than 800 items of product B are required per week. If the product A costs Rs. 5 per item and can be sold for Rs. 10 and product B costs Rs. 5 per item and can be sold for Rs. 8 per item, determine by simplex method how many items per week be produced for A and B in order to maximise the profit. Also write the dual of the problem. **[B. Com. (H) 2005(R)]**

Q10. The following information is available for a firm which manufacturers three products A, B, C, which are processed in two departments-Assembly and Finishing.

Product	Time required (in hours)		Profit per Unit (Rs.)
	Assembly	Finishing	
A	10	2	80
B	4	5	60
C	5	4	30
Capacity	2000	1000	

(i) Formulate the above problem as a linear programming problem.

(ii) Obtain optimal solution to the problem by using the simplex method. Which of the three products shall not be produced by the firm? Why

(iii) What are the shadow prices of the resources? **[B. Com. (H) 1993, 2006(C)]**

Q11. A company produces three products P_1, P_2 and P_3 from two raw materials A and B and Labour L. One unit of product P_1 requires one unit of A, 3 units of B and 2 units of L. A unit of product P_2 requires 2 units of A and B each, and 3 units of L, while one unit of P_3 needs 2 units of A, 6 units of B and 4 units of L. It is further known that the unit contribution margin for the products is Rs. 3, 2 and 5 respectively for P_1, P_2 and P_3.

Formulate this problem as a linear programming problem, and solve it to determine the optimal product mix. Is the solution obtained by you unique? Identify as alternate optimal solution, if any.

Also, obtain the shadow prices of the resources. **[B. Com. (H) 1994]**

Q12. 'X', A boat manufacturing company, makes three different kinds of boats. All can be made profitably in this company, but the company's production is constrained by the limited amount of labour, wood and screws available each month. The director will choose the combination of boats that maximizes his revenue in view of the information given in the following table:

Input	***Row Boat***	***Canoe***	***Kayak***	***Monthly Availability***
Labour	12	7	9	1260 hours
Wood Board (feet)	22	18	16	19008
Screws (kg)	2	4	3	396 kg
Selling Price in Rs.	4,000	2,000	5,000	

(i) Formulate the above as a linear programming problem.
(ii) Solve it by Simplex method.
(iii) Which, if any of the resources are not fully utilized? If so, how much is the spare capacity left?
(iv) What are the shadow prices?
(v) State the dual of the formulated problem. **[B. Com. (H) 2001, 2007(R)]**

Q13. A company produces two types of leather belts – Type A and Type B. The contribution to profit per belt is Rs. 8 for type A and Rs. 6 for type B. The time requirement of one belt of type A and type B are in the ratio of 2 : 3. Time available is sufficient to produce 500 belts of type A. The leather is sufficient for only 400 belts. Belt A require a fancy buckle and only 200 fancy buckles are available : **[B. Com. (H) 2007(C)]**
(i) Formulate the above as LPP.
(ii) Solve it by Simplex method.

Q14. A firm manufacturing office furniture provides you the following regarding resource consumption and availability and profit contribution :

Resources	Usage per unit			Availability
	Tables	Chairs	Books Case	
Timber (cu. ft.)	8	4	3	640
Assembly Deptt. (man hours)	4	6	2	540
Finishing Deptt. (man hours)	1	1	1	100
Profit Contribution per unit Rs.	30	20	12	

The firm wants to determine its optimal product mix : **[B. Com. (H) 2008]**
(i) Formulate the L.P.P. and solve with the simplex method.
(ii) Find the optimal product mix and the total maximum profit contribution.

Q15. Find basic solutions for the following equations :
$x + y + 2z = 6$
$3x + 2y + z = 10$
Which of these solutions are basic feasible solutions? **[B. Com. (H) 2010]**

Answers of Exercise 3

1. $x_1 = 24$ and $x_2 = 14$ for maximum value of z = 2200,
2. $x_1 = 20$ and $x_2 = 20$ for maximum value of z = 140,
3. $x_1 = 142$, $x_2 = 145$ and $x_3 = 0$ for maximum value of z = 20,060,
4. $x_1 = 1$, $x_2 = 1$, $x_3 = 1/2$ and $x_4 = 0$ for maximum value of Z = 13/2,
5. $x_1 = 1$, $x_2 = 1$, $x_3 = 1/2$ and $x_4 = 0$ for maximum value of Z = 13/2,
6.(i) S_1, X_2 and S_3, (ii) 3, 5, 0, 0, 0, (iii) No,
7. x = 6, y = 12 and max. profit is Rs. 132,
8. $x_1 = 0$, $x_2 = 6$ and max. value of z is 24,
9. 1 unit of product A and 0 units of product B are produced for maximum profit of Rs. 5,

10. 125 units of product A, 150 units of product B and 0 unit of product C are produced for maximum profit of Rs. 19,000. Shadow prices are 7 and 5,

11. 4 units of product P_1, 0 units of product P_2 and 0 units of product P_3 are produced. Shadow prices are 0, 1 and 0 respectively,

12. 12 row boats, 0 canoe and 124 kayak are produced monthly. Shadow prices are 1000/9, 0 and 4000/3 respectively

13. Max. Z = 8x + 6y, Subject to $2x + 3y \leq 100$, $x + y \leq 400$, $x \leq 200$, $x, y \geq 0$, and Solution is x = 200, y = 200 max Z = 2800

14.(i) Max $Z = 30x_1 + 20x_2 + 12x_3$, Subject to $8x_1 + 4x_2 + 3x_3 \leq 640$, $4x_1 + 6x_2 + 2x_3 \leq 540$ $x_1 + x_2 + x_3 \leq 100$, $x_1, x_2, x_3, \geq 0$.

(ii) $x_1 = 60$, $x_2 = 40$, $x_3 = 0$ and Max. Z = 2600

15. Basic solutions are (0, 14/3, 2/3), (14/5, 0, 8/5) and (-2, 8, 0) except (-2, 8, 0) other two are basic feasible solutions

Basic Concepts

1. **Unbounded Solution :** It occurs when there is no constraint on the solution so that one or more of the decision variables can be increased indefinitely without violating any of the restrictions. It occurs in maximization problems. In simplex method it is discovered if at any iteration stage, all the entries in minimum ratio column are either negative or infinite(∞). Thus, the rule for choosing the departing variable will not work. We can then stop the computations and conclude that the problem has an unbounded solution.

Exercise 4

Q1. Maximize Z = 3x + 3y, if possible,
subject to the constraints
$x - y \leq 1$, $x + y \geq 3$, $x, y \geq 0$

Q2. Maximize $Z = 3x_1 + 4x_2$
subject to the constraints
$-x_1 + x_2 \leq 5$, $-3x_1 + x_2 \leq 10$, $x_1, x_2 \geq 0$

Q3. Maximize $Z = 2x_1 + 5x_2$
subject to the constraints
$-4x_1 + x_2 \leq 5$, $-x_1 + x_2 \leq 4$, $x_1, x_2 \geq 0$

Q4. Maximize $Z = 5x_1 + 6x_2 + x_3$
subject to the constraints
$9x_1 + 3x_2 - 2x_3 \leq 5$, $4x_1 + 2x_2 - x_3 \leq 2$, $x_1 - 4x_2 + x_3 \leq 3$, $x_1, x_2, x_3 \geq 0$

Basic Concepts

1. **Multiple Optimal Solutions :** A linear programming problem having more than one optimal solution is said to have multiple solutions. This situation occurs when the objective function is parallel to one of the constraints, so the objective function will assume the same optimal solution at more than one solution point. For this reason they are called multiple optimal solutions. In simplex method, it is

discovered if in the final tableau a non-basic variable has 0 in the $C_j - Z_j$ row. The other optimal solutions are usually obtained by performing additional iterations of the simplex method, each time choosing a non-basic variable with zero $C_j - Z_j$ entry as the entering variable.

Exercise 5

Q1. Maximize Z = 4x + 8y, subject to constraints
$2x + y \le 0$, $x + 2y \le 24$, $x \ge 3$, $y \le 9$, $y \ge 0$

Q2. Maximize $Z = 9x_1 + 6x_2$, subject to the constraints
$x_1 + 2x_2 \le 8$, $3x_1 + 2x_2 \le 12$, $x_1, x_2 \ge 0$

Q3. Maximize $Z = 4x_1 + 10x_2$
subject to the constraints
$2x_1 + x_2 \le 50$, $2x_1 + 5x_2 \le 100$, $2x_1 + 3x_2 \le 90$, $x_1, x_2 \ge 0$

Q4. Maximize $Z = 6x_1 + 10x_2 + 2x_3$
subject to the constraints
$2x_1 + 4x_2 + 3x_3 \le 40$, $x_1 + x_2 \le 10$, $2x_2 + x_3 \le 12$, $x_1, x_2, x_3 \ge 0$

Q5. Given below is a table obtained after few iterations use Simplex method to solve a linear programming problem to maximize total contribution from the product A and product B.

Basic Variables	x_1	x_2	s_1	s_2	s_3	*Quantity*
x_2	0	1	3/5	-2/5	0	300
x_1	1	0	-2/5	3/5	0	300
s_3	0	0	-1/5	-1/5	1	400
C_j	8.5	10.5	0	0	0	

Give short answers to the following giving reasons:
(i) Is the above solution optimal?
(ii) Is the above solution feasible?
(iii) Does the problem have alternative solution? If so, find the other solution.
(iv) What are the shadow price of the resources? **[B. Com (H) 1998]**

Q6. A firm produces three products A, B, C using three resources (material, machine hours and labour hours). The manager of the firm wants to find out the best production strategy. A student of B. Com (Hons.) who was familiar with linear programming technique offered to help him. The student formulated the problem and solved the problem by Simplex method. He gave the following solution :

Contribution	x_1	x_2	x_3	S_1	S_2	S_3	**Quantity**
	30	40	10	0	0	0	
	½	1	3/2	1	0	0	45
	3/2	0	-1/2	-1/2	1	0	15
	5/2	0	½	-1/2	0	1	35

Do you agree that this is the best production strategy? If not, improve the solution to get the best production strategy. **[B. Com. (H) 2010]**
Also answer the following questions:

(i) Are all the three resources completely used? If not, how much of which resource is unused?
(ii) Can there be an alternative product mix which gives the same total contribution?
(iii) what are the marginal worth of the resources?
(iv) What happens if 15 machine hours are lost due to some mechanical problem.

Q7. A firm produces three products A, B and C using three resources, material, machine hours and labour hours. The manager intends to determine the ideal production strategy. A formulated the problem and solved it by Simplex method. He presented the following table : **[B. Com. (H) III Sem. 2012]**

C_j	30	40	10	0	0	0
Quantity	x_1	x_2	x_3	S_1	S_2	S_3
45	1/2	1	3/2	1	0	0
15	3/2	0	-1/2	-1/2	1	0
35	5/2	0	-1/2	-1/2	0	1

Complete the above table and answer the following questions:
(i) Do you agree that this is the ideal production strategy? If not, improve the strategy.
(ii) Are all resources fully used? If not, how much of each resource remains unused?
(iii) Can there be an alternative product mix that gives the same total contribution?
(iv) what are the shadow prices of resources?

Answers of Exercise 5

1. Maximum Z = 96 at either x = 12 and y = 6 or x = 6 and y = 9,
2. Maximum Z = 36 at either $x_1 = 4$ and $x_2 = 0$ or $x_1 = 2$ and $x_2 = 3$,
3. Maximum Z = 200 at either $x_1 = 0$ and $x_2 = 20$ or $x_1 = 75/4$ and $x_2 = 25/2$,
4. Max. Z = 84 at either $x_1 = 4$, $x_2 = 6$ and $x_3 = 0$ or $x_1 = 6$, $x_2 = 4$, and $x_3 = 4$,
5.(i) Yes, (ii) Yes, (iii) No, (iv) shadow prices are Rs. 2.9, Rs. 0.9 and Rs. 0 respectively
6. No, Best strategy is $x_1 = 10$, $x_2 = 40$, $x_3 = 0$ & max. z = 1900, (i) S_2 & S_3 are unused and unused capacity is 15 and 35 units respectively, (ii) Yes, $x_1 = 0$, $x_2 = 47.5$, $x_3 = 0$, (iii) $S_1 = 20$, $S_2 = 0$, $S_3 = 0$

Basic Concepts

1. **Degeneracy :** Degeneracy is another situation that can occur while solving an LP problem. Degeneracy in a LPP occurs when one or more of the basic variables assume a value of zero. In simplex method it is discovered when the replacement ratio calculations are made and there occurs a tie for the smallest ratio. To resolve it one can select any row arbitrarily but as a standard practice, select the lower of the two rows to find the departing variable.

Exercise 6

Q1. Maximize Z = $3x_1 + 9x_2$
subject to the constraints
$x_1 + 4x_2 \le 8$, $x_1 + 2x_2 \le 4$, $x_1, x_2 \ge 0$

Q2. Maximize Z = $80x_1 + 100x_2$
subject to the constraints
$20x_1 + 30x_2 \le 3$, $60x_1 + 40x_2 \le 4$, $x_1, x_2 \ge 0$

Q3. Maximise Z = $28x_1 + 30x_2$
Subject to
$6x_1 + 3x_2 \le 18$; $3x_1 + x_2 \le 8$; $4x_1 + 5x_2 \le 30$; $x_1, x_2 \ge 0$

Answers of Exercise 6

1. Maximum Z = 18 at $x_1 = 0$ and $x_2 = 2$,
2. Maximum Z = 10 at $x_1 = 0$ and $x_2 = 1/10$
3. Maximum Z = 180 at $x_1 = 0$ and $x_2 = 6$

Basic Concepts

1. **Use of Artificial Variables :** There are situations in maximization problems where the constraints have either of ≥ or = signs or both. In these problems the slack variables can not provide the solutions. In order to obtain an initial basic feasible solution, we first put the given LPP into its standard form and then a non negative variable is added to the left side of each of equation that lacks the much needed starting basic variables. Such a variable is called an artificial variable and plays the same role as a slack variable in providing the initial basic feasible solution. However, since such artificial variables have no meaning by virtue of original problem, so they must be forced out when optimum solution is attained.

2. **Big –M Method or Method of Penalties :** The Big-M method is used in solving linear programming problems involving artificial variables. The method is explained clearly in the following steps :

   (i) Write the LPP into its standard form and check whether there exists a starting basic feasible solution.

   (ii) If there does not exist a ready starting basic feasible solution, add artificial variables to the left side of each equation that has no slack variables or have slack variables with negative coefficients.

   (iii) Assign a very high penalty (-M for maximization problem and +M for minimization problem, M > 0).

   (iv) Apply the usual solution of the simplex method of obtaining optimum solution. By having a high penalty cost it is ensured that they will not appear in the final solution.

Exercise 7

Q1. Solve the following linear programming problem by using simplex method:
Maximise $z = 8x + 12y$ **[B. Com (H) 2003]**
Subject to : $x + y = 5$, $y \le 4$, $x \ge 2$, $x, y \ge 0$

Q2. Maximise Z = $2x_1 - x_2$
Subject to the constraints:
$3x_1 + x_2 = 3$, $4x_1 + 3x_2 \geq 6$, $x_1 + 2x_2 \leq 4$, $x_1, x_2 \geq 0$

Q3. Maximise Z = $x_1 + 2x_2 + x_3$
Subject to the constraints:
$x_1 - x_2 + x_3 \geq 4$, $x_1 + 2x_2 + 2x_3 \leq 8$, $x_1 - x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$

Q4. Maximise Z = $2x_1 + 3x_2 + 4x_3$
Subject to the constraints:
$3x_1 + x_2 + 6x_3 \leq 600$, $2x_1 + 4x_2 + 2x_3 \geq 480$, $2x_1 + 3x_2 + 3x_3 = 540$,
$x_1, x_2, x_3 \geq 0$

Q5. Maximise Z = $2x_1 + 3x_2 + x_3$ **[B. Com (H) 2005(R)]**
Subject to the constraints :
$4x_1 + 3x_2 + x_3 = 6$, $x_1 + 2x_2 + 5x_3 = 4$, $x_1, x_2, x_3 \geq 0$

Q6. A firm is planning to advertise a special sale on radio and television during a particular week. A maximum budget of Rs. 16,000 is approved for this purpose. It is known that radio commercials cost Rs. 1,000 per 30 second spot with a minimum contract of four spots, which the firm intends to enter. Television commercials on the other hand, cost Rs. 4000 per spot. Because of heavy demand, at the most 4 television spots are available in the week. Also, it is believed that a TV spot is five times as effective as a radio spot in reaching consumers. How should the firm allocate its advertising to attract the largest number of buyers. How will the optimal solution be affected if the availability of TV spots in not constrained? **[B. Com (H) 1993, 2006(C)]**

Q7. XYZ Company during the festival season combines two items A and B to form gift packs. Each pack must weigh 5 kg and should contain at least 2 kg of A and not more than 4 kg of B. The Company wants to determine the optimum mix. Formulate the above as a linear programming problem to maximize net contribution per pack and solve the same by using simplex method. **[B. Com (H) 1996]**

Q8. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each economy class ticket. The airline reserves at least 20 seats for first class. However, at least 4 times as many passengers prefer a to travel by economy class to the first class. Formulate as LPP and solve by simplex method to determine how many each type of tickets much be sold in order to maximize the profit for the airline. What is the maximum profit?

Q9. Given the following initial simplex table :

C_j			30	20	0	0	–M	–M
	B. V.	**Quantity**	x_1	x_2	S_1	S_2	A_1	A_2
0	S_1	8	1	1	1	0	0	0
–M	A_1	12	6	4	0	–1	1	0
–M	A_2	20	5	8	0	0	0	1
	Z_j	–32M	–11M	–12M	0	M	–M	–M
	$C_j - Z_j$		30+11M	20+12M	0	M	0	0

Write down the original problem represented by the above table. Find the optimum solution. **[B. Com. (H) 2010]**

Answers of Exercise 7

1. $x = 2$, $y = 3$ and Max. Z = 52, **2.** $x_1 = 3/5$, $x_2 = 6/5$, Max. Z = 0,

3. $x_1 = 18/5$, $x_2 = 6/5$ and $x_3 = 8/5$ and Max. value of z = 54/5,

4. $x_1 = 0$, $x_2 = 96$ and $x_3 = 84$ and Max. value of z = 624,

5. $x_1 = 0$, $x_2 = 2$ and $x_3 = 0$ and Max. value of z = 6,

6. 4 TV Spots, 3 Radio Spots, **7.** Unbounded Solution,

8. Maximum profit is Rs. 64,000 at 40 first class and 160 economy class tickets.

9. Max.Z = $30x_1 + 20x_2$ Subject to $x_1 + x_2 \leq 8$, $6x_1 + 4x_2 \geq 12$, $5x_1 + 8x_2 = 20$, $x_1, x_2 \geq 0$
Solution : $x_1 = 4$, $x_2 = 0$ & Max. Z = 120

Basic Concepts

1. **Infeasibility :** Infeasibility is said to exist when a given problem has no feasible solution, and is evident graphically when no common point is found in the feasible egions of all the constraints of a problem taken jointly. However, graphical method cannot be used for more than two variables, in which case simplex method is appropriate. In simplex method when in the final solution, an artificial variable is in the the basis at a positive value then there is no feasible solution to the problem.

Exercise 8

Q1. Maximise Z = $20x_1 + 30x_2$
Subject to $2x_1 + x_2 \leq 40$; $4x_1 - x_2 \leq 20$; $x_1 \geq 30$; $x_1, x_2 \geq 0$

Q2. Maximise Z = $3x_1 + 4x_2$
Subject to
$2x_1 + x_2 \leq 10$; $x_1 + 4x_2 \leq 36$; $x_1 + 2x_2 \leq 10$; $x_1 \geq 5$; $x_2 \geq 7$

Q3. A firm manufactures two products A and B, which pass through manual and machine precesses. The time required by each unit and the total available time is as given here :

	Per unit requirement (hours)		Total hours available
	A	B	
Labour	4	2	1600
Machine	6	5	3000

The enthusiastic production manager requires that at least 300 units of A and 300 units of B should be produced. It is known that one unit of A yields Rs. 10 of profit, while a unit of B yields Rs. 8. In your opinion, what should be the optimum production mix so that all the constraints are satisfied?

Q4. From the following initial simplex tableau :

Basic variable	C_j	x_1 15	x_2 25	S_1 0	S_2 0	A_1 –M	A_2 –M	
A_1	–M	7	6	–1	0	1	0	20
S_2	0	8	5	0	1	0	0	30
A_2	–M	3	–2	0	0	0	1	18
Z_j		–10M	–4M	M	0	–M	–M	–38M
$C_j - Z_j$		15+10M	25 + 4M	–M	0	0	0	

(i) Write down the original problem represented by the above tableay.
(ii) Find out the optimal solution of this problem.
(iii) Is it a unique solution? Why? **[M. Com. 1996]**

Answers of Exercise 8

1. Infeasible Solution, **2.** Infeasible Solution, **3.** Infeasible Solution

4(i). Max. $Z = 15x_1 + 25x_2$

Subject to : $7x_1 + 6x_2 \geq 20$; $8x_1 + 5x_2 \leq 30$; $3x_1 - 2x_2 = 18$; $x_1, x_2 \geq 0$,

(ii) Infeasible Solution,

Basic Concepts

1. **Solution of Minimization Problems by Simplex Method :** The following steps are taken to solve a minimization problem by LPP :

(i) Express the LPP in the standard form by subtracting surplus variables and adding artificial variables.

(ii) Assign a zero coefficient to the surplus variable and large penalty value M to the artificial variable in the objective function.

(iii) Set up the initial simplex tableau.

(iv) Compute the elements of $C_i - Z_i$, *i.e.*, index row.

(v) If all the elements of $C_i - Z_i$ row are greater than or equal to zero, then the current basic feasible solution is optimal.

(vi) If at least one $C_i - Z_i \leq 0$, then select the most negative number in the index row. The column is known as the pivot column, and determines the entering variable to the solution basis in the next simplex tableau.

(vii) Determine the pivot row and number in the same way as discussed in the maximization problems.

(viii) Set up the next simplex tableau and move back to Step (iv).

Exercise 9

Q1. Minimize $Z = x - 5y + 20$, subject to the constraints

$x - y \geq 0$, $-x + 2y \geq 2$, $x \geq 3$, $y \leq 4$, $x, y \geq 0$

Q2. Minimize $Z = 2x + 3y$

Subject to the constraints

$x + y \geq 1$, $10x + y \geq 5$, $x + 10y \geq 1$, $x, y \geq 0$.

Q3. Minimise $Z = 4x_1 + x_2$

Subject to the constraints:

$3x_1 + 4x_2 \geq 20$, $-x_1 - 5x_2 \leq -15$, $x_1, x_2, \geq 0$

Q4. Minimise $Z = 2x_1 + 5x_2 + 3x_3$

Subject to the constraints:

$3x_1 + x_3 \geq 600$, $5x_1 + x_2 + 2x_3 \geq 15$, $x_1 + x_2 \geq 8$, $x_1, x_2, x_3 \geq 0$

Q5. The following is the initial table of a linear programming problem:

	X_1	X_2	S_1	S_2	A_1	A_2	Quantity
	3	1	-1	0	1	0	27
	1	1	0	0	0	1	21
	1	2	0	1	0	0	40
C_j	8	4	0	0	M	M	

(i) Write the corresponding linear programming problem.
(ii) Solve it by simplex method.
(iii) Is it a case of multiple solution, if yes also find the alternative solution.
(iv) Write its dual and read the optimum solution of dual. **[B. Com (H) 2005(C)]**

Q6. The standard weight of a special purpose brick is 5 kg and it contains two basic ingredients B_1 and B_2. B_1 costs Rs. 5 per kg and B_2 costs Rs. 8 per kg. Strength considerations dictate that the brick contains not more than 4 kg of B_1 and minimum of 2 kg of B_2. Since the demand for the product is likely to be related to the price of the brick, find the minimum cost of brick satisfying the above conditions. **[B. Com (H) 1997]**

Q7. A finished product must weigh exactly 150 gm. The two raw materials used in manufacturing the product are A with a cost of Rs. 2 per unit and B with a cost of Rs. 8 per unit. At least 14 units of B and not more then 20 units of A must be used. Each unit of A and B weighs 5 and 10 gm respectively. How much of each type of raw material should be used for each unit of final product to minimize the cost. **[B. Com (H) 2005(C)]**

Q8. XYZ Chemical Corporation must produce exactly 1,000 kg of a special mixture of Chemical A and Chemical B for a customer. Chemical A costs ` 50 per kg and Chemical B costs ` 60 per kg. No more than 300 kg of Chemical A can be used and at least 150 kg of Chemical B must be used. Using simplex method, determine the least cost blend of the two ingredients. **[B. Com. (H) 2012]**

Answers of Exercise 9

1. Minimum Z = 4 at $x = 4$, $y = 4$, **2.** Z = 2 at $x = 1$, $y = 0$,
3. $x_1 = 0$ and $x_2 = 5$ and Min. value of Z = 5,
4. $x_1 = 184/9$, $x_2 = 0$ and $x_3 = 0$ and Min. value of z = 368/9,
5. $x_1 = 3$, $x_2 = 18$ and minimum value of z is 96,
6. 3 kg of B_1 and 2 kg of B_2 are used for minimum cost of Rs. 31,
7. 2 units of A and 14 units of B are used for minimum cost of Rs. 116,
8. Chemical A 300 kg, Chemical B 700 kg, least cost ` 57,000

Basic Concepts

1. **Duality Theory :** For every LPP, there is a related unique another LPP involving the same data which also describes original problem. The given original problem is called *Primal*, and the related other problem is called *Dual*, which can be obtained by transposing the rows and columns of the primal. A solution of dual may also be obtained similar to that of primal. The two problems have very closely related properties so that optimal solution of the dual gives complete information about the optimal solution of the primal and vice versa.

2. **Steps For Constructing the Dual from Primal :** The following steps are required for construction of a dual :
   (i) If the primal is a maximization problem then make dual as minimization problem and vice versa.

(ii) The number of variables in the dual are equal to the number of constraints in the primal and the number of constraints in the dual are equal to the number of variables in the primal.

(iii) The coefficients of the objective function in the dual problem will come from the right hand side of the primal.

(iv) The coefficients of each constraint in the dual will come from the coefficients of respective variable in the primal.

(v) If the ith constraint in the primal is an equation then the ith variable in the dual will be unrestricted.

(vi) if the ith variable in the primal is unrestricted then the ith constraint in the dual is equation.

3. **Solution of Minimization Problems by Dual :** Solution of a minimization problem by taking its dual is more simplified than solving it by using artificial variables. The solution of the minimization problem may be read from the final table of its dual.

4. **Economic Interpretation of Duality :** The simplex method provides more than just the optimal solution. It provides additional information which can be useful for managerial decision making, specifically, the shadow price for each resource. The solution of the variables of the dual represents the *shadow prices* of the resources.
An existence of a slack variable in the final simplex variable tableau of the primal indicates that the resource has not been fully utilized. Non existence of slack variable indicate full utilization of the existing resources.

Exercise 10

Q1. Write the dual to the following LPP.
Minimise $Z = 16x_1 + 9x_2 + 21x_3$
Subject to :
$x_1 + x_2 + 3x_3 \geq 16$, $2x_1 + x_2 + x_3 \geq 12$, $x_1, x_2, x_3 \geq 0$

Q2. Obtain the dual of :
Max. $z = 3x_1 + 4x_2 + x_3$
Subject to the constraints :
$x_1 + 2x_2 + 3x_3 \leq 90$, $2x_1 + x_2 + x_3 \leq 60$, $3x_1 + x_2 + 2x_3 \leq 80$
$x_1, x_2, x_3 \geq 0$

Q3. Write the dual programme for the following linear programming problem :
Maximize : $z = 3x_1 + 4x_2 + 7x_3$
Subject to the constraints :
$x_1 + x_2 + 3x_3 \leq 10$, $4x_1 - x_2 + 2x_3 \geq 15$
$x_1, x_2, \geq 0$; x_3 unrestricted variable **[B. Com. (H) 2011]**

Q4. Write the dual to the following LPP.
Max. $z = 20x_1 + 15x_2 + 18x_3 + 10x_4$
Subject to :
$4x_1 - 3x_2 + 10x_3 + 4x_4 \leq 60$, $x_1 + x_2 + x_3 = 27$, $-x_2 + 4x_3 + 7x_4 \geq 35$
$x_1, x_2, x_3 \geq 0$ x_4 : unrestricted in sign **[B. Com (H) 2000, 2006(C)]**

Q5. Find the dual of the following linear programming problem :

Maximize : $z = 7x_1 + 8x_2 + 6x_3$

Subject to the constraints :

$x_1 + 4x_2 + 4x_3 = 8,\qquad 3x_1 + 5x_2 + 3x_3 \le 12$

$x_1, x_2, x_3 \ge 0$ **[B. Com. (H) 2012]**

Q6. Write the dual of the following problem. **[B. Com (H) 2008]**

Max. $z = 4x_1 + 2x_2 + 3x_3$

Subject to :

$x_1 + 2x_2 \ge 3,\qquad x_2 - 3x_3 \ge 6,\qquad -x_1 + 3x_2 - 2x_3 \le 3,\qquad x_1, x_2, x_3 \ge 0$

Q7. Find the dual of the following : **[B. Com (H) 2009]**

Minimise $Z = 5x_1 - 6x_2 + 4x_3$

Subject to

$3x_1 + 4x_2 + 6x_3 \ge 9$; $\quad x_1 + 3x_2 + 2x_3 \ge 5$; $\quad 7x_1 - 2x_2 - x_3 \le 10$;

$x_1 - 2x_2 + 4x_3 \ge 4$; $\quad 2x_1 + 5x_2 - 3x_3 = 3$; $\quad x_1, x_2, x_3 \ge 0$

Q8. The following information is available for a firm which manufacturers three products A, B, C, which are processed in two departments-Assembly and Finishing.

Product	Time required (in hours)		Profit per Unit (Rs.)
	Assembly	Finishing	
A	10	2	80
B	4	5	60
C	5	4	30
Capacity	2000	1009	

(i) Formulate the above problem as a linear programming problem.

(ii) Obtain optimal solution to the problem by using the simplex method. Which of the three products shall not be produced by the firm? Why

(iii) Write the dual to the above problem and determine the optional values of the dual variables

(iv) What are the shadow prices of the resources? **[B. Com (H) 1993]**

Q9. A firm uses three machines in the manufacture of three products. Each unit of product A requires 3 hours on machine 1, 2 hours on machine II and one hour on machine II and 3 hours on machine III, while each unit of product C requires 2 hours on each of the three machines, The contribution margin of the three products is Rs. 30, Rs. 40 and Rs. 35 per unit respectively. The machine hours available on the three machines are 90, 54 and 93 respectively.

(i) Formulate the above as a linear programming problem and solve for maximum profit, using simplex method.

(ii) Write the dual to the LPP.

(iii) Obtain the optimum values of the dual variables and verify that the primal and the dual problems have the same objective function values. **[B. Com (H) 1995]**

Q10. A firm has two grades of cashew nuts: Grade I- 750 and Grade II-1200 kg. These are to be mixed in two types of packets of 1kg each-economy and special. The economy pack consists of Grade I and Grade II in the production of 1:3, while the special pack combines the two in equal proportion. The contributions of the economy and the special packs are Rs. 5 and Rs. 8 per pack respectively.

(i) Formulate this as a linear programming problem to maximize contribution and solve it by simplex method.

(ii) Write the dual of the above problem, read the optimum solution of the dual and give its economic interpretation. **[B. Com (H) 1998]**

Q11. A company produces 2 types of leather belts-Types A and B. Contribution per belt is Rs. 4 for type A and Rs. 3 for type B. The time requirements of one belt of type A and type B are in the ratio of 2 : 3. Time available is sufficient to produce 500 belts of type A. The leather is sufficient for only 400 belts. Belt A requires a fancy buckle and only 200 fancy buckles are available.

(i) Formulate above as a linear programming problem.

(ii) Solve it by simplex method and comment on this optimum solution.

(iii) Write its dual and read the optimum solution of the dual. **[B. Com (H) 1999]**

Q12. A manufacturer makes three types of decorative tensor lamps; model 1200, model 1201 and model 1202. The requirement of raw materials for all lamps is the same, while the production time for each differs. Each model 1200 lamp requires 0.1 hr of assembly time, 0.2 hr of writing time, and 0.1 hr of packaging time. The model 1201 requires 0.2 hr of assembly, 0.3 hr of wiring time, and 0.1 hr of packaging time. The model 1202 requires 0.3 hr of assembly, 0.4 hr of wiring time, and 0.1 hr of packaging time. The manufacturer makes a profit of Rs. 120 on each model 1200 lamp, Rs. 190 on each model 1201 lamp, and Rs. 210 on each model 1202 lamp. The manufacturer can schedule up to 80 hrs of assembly labour, 120 hrs of wiring labour, and 100 hrs of packaging labour. Assuming that all lamps can be sold, determine the optimal quantities of each model and the marginal values of each of the resources.

Write the dual of the given problem and obtain optimal values of the dual variables. **[B. Com (H) 2000]**

Q13. A firm buys casting of P and Q type of parts and sells them as finished products after machining, boring and polishing. The purchasing costs for casting are Rs. 3 and Rs. 4 each for parts P and Q respectively and selling prices are Rs. 8 and Rs.10 respectively. The per hour capacity of machines used for machining, boring and polishing for the two products is given below:

	Capacity (per hour)		
	P	or	Q
Machining	30	or	50
Boring	30	or	45
Polishing	45	or	30

The running costs for machining, boring and polishing are Rs. 30, Rs. 22.5 and Rs. 22.5 per hour respectively. Formulate the linear programming problem to find out the product mix to maximize the profit and solve the same using simplex method. Also write its dual and read the optimum solution of dual from the optimum solution of primal. **[B. Com (H) 2004]**

[**Hint :** Profit = SP – Purchasing cost – Running Cost]

Q14. A horticulturist wishes to mix fertilizer brands that will provide a minimum of 15 units of potash, 20 units of nitrates, and 24 units of phosphates. Brand 1 provides 3 units of potash, 1 unit of nitrates and 3 units of phosphates; it costs Rs. 120. Brand 2 provides 1 unit of potash, 5 units of nitrates and 2 units of phosphates; it costs Rs. 60. **[B.B.E. 2010]**

(i) Find the least cost combination of fertilizers that will meet the desired specifications.

(ii) Write the dual problem.

(iii) Which of the variables of the dual shall have a zero optimal value ? Why ?

Q15. A firm produces and sells two products A and B. The profit per unit of A is Rs. 40 and per unit of B is Rs. 30. The products are processed on the same machine but sold in two different markets. It takes three times of machine time to produce a unit of A as compared to a unit of B, and, if all time is devoted to producing A, the machine can produce a maximum of 10,000 units. The market research indicates that the firm can sell a maximum of 8,000 units of A and 15,000 units of B.

(i) Formulate the above as a linear programming problem to maximize profit and solve the same by simplex method.

(ii) Write the dual of the problem as formulated above and read the optimum values of the variables of the dual from the optimum table of the solution of primal. **[B. Com. (H) III Sem. 2012]**

(iii) Verify that the objective function values of primal and dual are equal.

Solution of Minimization Problems by Dual

Q16. Solve by simplex method using duality :

Min. $z = 5x_1 + 4x_2 + 3x_3$

Subject to the constraints :

$x_1 + x_2 + x_3 \geq 100$, $2x_1 + x_2 \geq 50$, $x_1, x_2, x_3 \geq 0$

Q17. A firm produces three types of biscuits A, B and C. It packs them in assortment of two sizes I and II . Size I contains 20 biscuits of type a, 50 of type B and 10 of type C. The size II contains 10 biscuits of type A, 80 of type B and 60 of type C. A buyer intends to buy at least 120 biscuits of type A, 740 of type B and 240 of type C. Determine the least number of packets he should buy.

Use simplex method and concept of dual. **[B. Com (H) 2006(R)]**

Q18. Vitamins A, B and C are found in foods F_1 and F_2. One units of F_1 contains 1mg of A, 100 mg of B and 10 mg of C. One unit of F_2 contains 1 mg A, 10 mg of B and 100 mg of C. The minimum daily requirements of A, B and C are 1 mg, 50 mg and 10 mg respectively. The costs per unit of F_1 and F_2 are Re.1 and Rs. 1.5 respectively. You are required to (i) formulate the above as a linear programming problem minimizing the cost per day, (ii) write the dual of the problem and (iii) solve the dual by using simplex method and read there from the answer to the primal. **[B. Com (H) 1992]**

Q19. A diet is to contain at least 20 ounces of protein and 15 ounces of carbohydrate. There are three foods A, B and C available in the market. Costing Rs. 2, Re. 1 and Rs. 3 per unit respectively. Each unit of A contains 2 ounces of protein and 4 ounces of carbohydrate. Each unit of B contains 3 ounces of protein and 2 ounces of carbohydrate. Each unit of C contains 4 ounces of protein and 2 ounces of carbohydrate. Formulate the LPP so as to minimize the cost of diet. Find its dual. Solve the dual by simplex method and read from dual solution the solution to the primal problem. **[B. Com. (H) 2007(R)]**

Q20. A manufacturing company makes three products, each of which requires three operations as part of the manufacturing process. The company can sell all of the products it can manufacture but its production capability is limited by the capacity of its operation centres. Additional data concerning the company are as follows :

Product	Manufacturing Requirement hours/unit			Cost Rs.	Selling Price Rs.
	Centre1	Centre 2	Centre3		
A	1	3	2	11	15
B	3	4	1	12	20
C	2	2	2	10	16
Hours avail.	160	120	80		

What should the product mix be? Write the dual of the given problem. Give its economic interpretation and use it for checking the optimal solution of the given problem. **[B. Com. (H) 2011]**

Q21. A person consumes two types of food A and B every day to obtain 8 units of protein, 12 units of carbohydrates and 9 units of fats which is his daily minimum requirement. 1 kilo of food A contains 2, 6 and 1 units of protein, carbohydrates and fats respectively. 1 kilo of food B contains 1, 1 and 3 units of protein, carbohydrates and fats respectively. Food A costs ` 8.50 per kilo and food B costs ` 4 per kilo. Determine how many kilos of each food he should buy to minimize his cost of food and still meet the minimum requirements. Formulate the problem mathematically. Write its dual and solve the dual by the simplex method. **[B. Com. (H) 2013(C)]**

Q22. A factory produces three different products A, B and C. The profit per unit of these products is Rs. 3, Rs. 4 and Rs. 6 respectively. The products are processed in three operations, viz., X, Y and Z and the time (in hours) required in each unit are given below :

	Products		
Operations	**A**	**B**	**C**
X	4	1	6
Y	5	3	1
Z	1	2	3

The factory has 3 machines for operation X, 2 machines for operation Y and only one machine for operation Z. The factory works 25 days in a month, at the rate of 16 hours a day in two shifts. The effective working of all the processes is only 80% due to power cuts or breakdown of machines. **[B Com (H) 2019]**

(i) Formulate the problem mathematically.

(ii) Use simplex method to find how many units of each product should be produced monthly in order to maximize profit.

(iii) Write the dual to the above problem and determine the optimum values of the dual variables from the primal.

Answers of Exercise 10

1. Max. Z = $16y_1 + 12y_2$
Subject to Constraints :
$y_1 + 2y_2 \le 16$, $y_1 + y_2 \le 9$, $3y_1 + y_2 \le 21$, $y_1, y_2 \ge 0$

2. Min. Z = $90y_1 + 60y_2 + 80y_3$
Subject to Constraints :
$y_1 + 2y_2 + 3y_3 \ge 3$, $2y_1 + y_2 + y_3 \ge 4$, $3y_1 + y_2 + 2y_3 \ge 1$, $y_1, y_2, y_3 \ge 0$,

3. Min. Z = $10y_1 - 15y_2$
Subject to the constraints :
$y_1 - 4y_2 \ge 3$, $y_1 + y_2 \ge 4$, $3y_1 - 2y_2 = 7$, $y_1, y_2 \ge 0$,

4. Min. Z = $60y_1 + 27y_2 + 35y_3$
Subject to Constraints :
$4y_1 + y_2 - y_3 \ge 20$, $-3y_1 + y_2 \ge 15$, $10y_1 + y_2 + 4y_3 \ge 18$, $4y_1 + 7y_3 = 10$, $y_1, y_3 \ge 0$, y_2 is unrestricted in sign,

5. Min. Z* = $8y_1 + 12y_2$ Subject to
$y_1 + 3y_2 \ge 7$, $4y_1 + 5y_2 \ge 8$, $4y_1 + 3y_2 \ge 6$, y_1 unrestricted in sign and $y_2 \ge 0$,

6. Min. Z = $3y_1 + 6y_2 - 3y_3$, subject to $y_1 + y_3 \le 4$, $2y_2 + y_2 - 3y_3 \le 2$, $-3y_2 + 2y_3 \le 3$, $y_1, y_2, y_3 \ge 0$,

7. Max. Z = $9y_1 + 5y_2 - 10y_3 + 4y_4 + 3y_5$
$3y_1 + y_2 - 7y_3 + y_4 + 2y_5 \le 5$; $4y_1 + 3y_2 + 2y_3 - 2y_4 + 5y_5 \le -6$; $6y_1 + 2y_2 + y_3 + 4y_4 - 3y_5 \le 4$; $y_1, y_2, y_3, y_4 \ge 0$ y_5 : unrestricted in sign,

8. 142 units of product A, 145 units of product B and 0 unit of product C are produced for maximum profit of Rs. 20,060. Shadow prices are 20/3 and 20/3,

9. 0 units of product A, 12 units of product B and 21 units of product C are produced for maximum profit of Rs. 1215. Shadow prices are 21/2, 10 and 0 respectively,

10. 900 units of economy pack and 1050 units of special pack are produced for maximum profit of Rs. 12,900,

11. 200 units of type A and 200 units of type B are produced for maximum profit of Rs. 1400,

12. 0 units of model 1200, 400 units of model 1201 and 0 units of model 1202 are produced for maximum profit of Rs. 76,000,

13. 0 units of part P and 30 units of part Q are produced for maximum profit of Rs. 30,

14.(i) $x_1 = 2$, $x_2 = 9$ and Min. Z = 780, **(ii)** Max. Z = $15y_1 + 20y_2 + 24y_3$ subject to $3y_1 + y_2 + 3y_3 \le 120$, $y_1 + 5y_2 + 2y_3 \le 60$, $y_1, y_2 \ge 0$, **(iii)** $y_1 = 20$, $y_2 = 0$, $y_3 = 20$,

15.(i) Max. Z = $40x_1 + 30x_2$ Subject to $3x_1 + x_2 \le 30{,}000$, $x_1 \le 8{,}000$, $x_2 \le 15{,}000$; maximum value at $x_1 = 5{,}000$, $x_2 = 15{,}000$ and max Z = 6,50,000
(ii)Min Z* = $30{,}000y_1 + 8{,}000y_2 + 15{,}000y_3$ Subject to $3y_1 + y_2 \ge 40$, $y_1 + y_3 \ge 30$,

16. $x_1 = 0$, $x_2 = 50$ and $x_3 = 50$ and Min. value of z = 350,

17. 2 packets of size I and 8 packets of size II are to be bought,

18. 1 unit of food F_1 and 0 unit of food F_2, minimum cost is Re. 1,

19. 0 units of Food A, 15/2 units of Food B, 0 units of Food C, Min. Z = 15/2

20. Max. Z = $4x_1 + 8x_2 + 6x_3$ Subject to $x_1 + 3x_2 + 2x_3 \le 160$, $3x_1 + 4x_2 + 2x_3 \le 120$, $2x_1 + x_2 + 2x_3 \le 80$, $x_1, x_2 \ge 0$; Solution $x_1 = 0$, $x_2 = 40/3$, $x_3 = 100/3$ and Max Z = 920/3

22.(i) Max. Z = $3x_1 + 4x_2 + 6x_3$ Subject to $4x_1 + x_2 + 6x_3 \le 960$, $5x_1 + 3x_2 + x_3 \le 640$, $x_1 + 2x_2 + 3x_3 \le 320$, $x_1, x_2, x_3 \ge 0$; **(ii)** $x_1 = 800/7$, $x_2 = 0$, $x_3 = 480/7$ and Max Z = 5280/7 **(iii)** Min Z = $960y_1 + 640y_2 + 320y_3$ Subject to $4y_1 + 5y_2 + y_3 \ge 3$, $y_1 + 3y_2 + 2y_3 \ge 4$, $6y_1 + y_2 + 3y_3 \ge 6$, $y_1, y_2, y_3 \ge 0$, Solution $y_1 = 0$, $y_2 = 3/14$, $y_3 = 27/14$.

Exercise 11

Miscellaneous Problems

Q1. While solving a linear programming problem by Simplex Method, how will you detect that :

(i) Problem has unbounded solution;
(ii) problem has multiple optimum solutions;
(iii) problem has degenerate solution;
(iv) problem is infeasible. **[B. Com. (H) 2010]**

Q2. Given below is a table appearing in the course of the simplex solution of a maximisation problem :

C_j	Basic variable	Solution Value	2 x_1	4 x_2	1 x_3	0 S_1	0 S_2	0 S_3	–M A_1
2	x_1	4	1	2	½	0	0	7/8	0
0	S_2	12	0	0	–1	0	1	–½	0
0	S_1	12	0	6	0	1	0	1	–1
	Z_j	8	2	4	1	0	0	7/4	0
	$C_j - Z_j$		0	0	0	0	0	–7/4	–M

Give short answers to the following questions, briefly stating the reasons :

(i) Is the solution indicated above optimal?
(ii) What is the optimal product mix and the total maximum profit?
(iii) Is the above solution degenerate?
(iv) Does the problem have multiple optimal solutions?
(v) Write alternative solution, if any.
(vi) What is the objective function of the problem? **[B. Com (H) 2008]**

Q3. The Simplex tableau for a maximization problem of linear programming is given below : **[B. Com. (H) 2009]**

C_j	4 x_1	5 x_2	0 S_1	0 S_2	*Quantity*
	1	1	1	0	10
	1	0	-1	1	3

Answer the following questions, giving reasons in brief :

(a) Is this solution optimal ?
(b) Is it a case of multiple solution ? If yes, find the alternative solution.
(c) Is this solution degenerate?
(d) Is this solution feasible?
(e) If s_1 is slack in machine A (in hours/week) and s_2 is slack in machine B (in hours/week), which one of these machines is being used to the full capacity when producing according to this solution.
(f) A customer would like to have one unit of product x_1 and is willing to pay in excess of the normal price in order to get it. How much should the price be increased in order to ensure no reduction of profits?
(g) How many units of two products x_1 and x_2 are being produced according to this solution, and what is the total profit.
(h) Machine A (associated with slack s_1, in hours/week) has to be shut down for repairs for 2 hours next week. What will be the effects on profits?
(i) How much would you be prepared to pay for another hour (per week) of capacity each on machine A and machine B?

Q4. Given below is the Simplex tableau for a maximization type of linear programming problem: **[B. Com. (H) 2011]**

$C_j \rightarrow$ $\downarrow$	Basic Variable	4 x_1	5 x_2	0 S_1	0 S_2	b_j
5	x_2	1	1	1	0	6
0	S_2	1	0	-1	1	2

Answer the following questions, with reasons :

(a) Does the tableau represent an optimal solution ?
(b) Are there more than one optimal solutions?
(c) Is this solution degenerate?
(d) Is this solution feasible?
(e) If s_1 is slack in machine A (in hours/week) and s_2 is slack in machine B (in hours/week), which one of these machines is being used to the full capacity when producing according to this solution.
(f) A customer would like to have one unit of product x_1 and is willing to pay in excess of the normal price in order to get it. How much should the price be increased in order to ensure no reduction of profits?
(g) How many units of two products x_1 and x_2 are being produced according to this solution, and what is the total profit.
(h) Machine A (associated with slack s_1, in hours/week) has to be shut down for repairs for 2 hours next week. How much will the reduction in profits be ?
(i) What is the maximum you would be prepared to pay for another hour (per week) of capacity each on machine A and machine B?
(j) What are the shadow prices of the machine hours ?

Q5. Given below is the simplex table for a maximization type of linear programming problem :

Cj → 10 6 4 0 0 0

Basic Variable	x_1	x_2	x_3	S_1	S_2	S_3	Quantity
x_2	0	1	$\frac{5}{6}$	$\frac{5}{3}$	$-\frac{1}{6}$	0	$\frac{200}{3}$
x_1	1	0	$\frac{1}{6}$	$-\frac{2}{3}$	$\frac{1}{6}$	0	$\frac{100}{3}$
S_3	0	0	4	-2	0	1	100

Answer the following questions with reasons :

(i) Does the table above gives an optimal solution?

(ii) Is this solution feasible?

(iii) Does the problem have alternative optimal solutions?

(iv) Is this solution degenerate?

(v) Write the optimal product mix and the profit contribution shown by the above solution.

(vi) Indicate the shadow prices of three resources.

(vii) If the company wishes to expand the production capacity, which of the three resources should be given prioirity?

(viii) If a customer is prepared to pay higher price for x_3, how much should the price be increased so that the company's profit remains unchanged?

(ix) Write down the objective function of the above problem.**[B. Com. (H) 2012]**

Q6. In a product mix problem, x_1, x_2, x_3 and x_4 indicate the units of products A, B, C, and D respectively and we have :

Maximise profit Z = $4x_1 + 6x_2 + 3x_3 + x_4$

Subject to :

$1.5x_1 + 2x_2 + 4x_3 + 3x_4 \leq 550$ (Machine I hours)

$4x_1 + x_2 + 2x_3 + x_4 \leq 700$ (Machine II hours)

$2x_1 + 3x_2 + x_3 + 2x_4 \leq 200$ (Machine III hours)

x_1, x_2, x_3 and $x_4 \geq 0$

(a) Solve it with simplex method and find out the optimal product-mix as well as the total maximum profit contribution.

(b) Is there any other optimal solution to the problem?

(c) Write the dual to the given LPP. Obtain the optimal values of the dual variables and verify that the primal and the dual problems have the same objective function values.

(d) What are the shadow prices of the machine hours on the three machines? If we wish to expand the production capacity, which of the three machines should be given priority?

(e) If a customer is prepared to pay a higher price for the product A, by how much should the price is increased so that the company's profit remains unchanged? **[M. Com. 2007]**

Answers of Exercise 11

2.(i) Yes, **(ii)** Max. profit = 8 when $x_1 = 4$, $x_2 = 0$, $x_3 = 0$, **(iii)** No,

3.(a) yes, **(b)** No, **(c)** No, **(d)** Yes, **(e)** A, **(f)** price of x_1 should be increased at least by Re. 1, **(g)** $x_1 = 0$, $x_2 = 10$ and Z = 50,

(h) profit will decrease by Rs. 10, **(i)** Rs. 5 and Rs. 0

4.(a) yes, **(b)** No, **(c)** No, **(d)** Yes, **(e)** A, **(f)** price of x_1 should be increased at least by Re. 1, **(g)** $x_1 = 0$, $x_2 = 6$ and Z = 30,

(h) profit will decrease by Rs. 10, **(i)** Rs. 5 and Rs. 0, **(j)** Rs. 5 and Rs. 0,

5.(i) Yes, **(ii)** Yes, **(iii)** No, **(iv)** No, **(v)** $x_1 = 100/3$, $x_2 = 200/3$, $x_3 = 0$, profit = 2200/3, **(vi)** 10/3, 2/3, 0, **(vii)** , **(viii)** 20/3, **(ix)** Max. $Z = 10x_1 + 6x_2 + 4x_3$,

6.(a) $x_1 = 0$, x_2 25, $x_3 = 125$, $x_4 = 0$ and Max. Z = 525, **(b)** No, **(c)** $y_1 = 3/10$, $y_2 = 0$, $y_3 = 9/5$ Min. Z* = 525, **(d)** Re. 3/10, Re. 0 & Re. 9/5 per hour respectively, M III, **(e)** 5 paise

Exercise 12

Theory Questions

Q1. In the context of linear programming, what do you mean by no optimal solution, multiple optimum solution and unbounded optimum solution? How would you recognize these in the optimum simplex tableau? **[B. Com. (H) 2002]**

Q2. In case of linear programming problems, what is a feasible solution and an infeasible solution? Describe each giving an imaginary graph. **[B. Com. (H) 2003]**

Q3. What do you mean by unrestricted variable in linear programming? How will you deal with this situation in simplex method? **[B. Com. (H) III Sem. 2012]**

Q4. "The duality is not a mere mathematical trick; the dual variables have managerial significance." Explain the statement with the help of an example. **[B. Com. (H) III Sem. 2012]**

Q5. In the context of linear programming, what do you mean by (i) unbounded solution, (ii) multiple optimal solutions, (iii) no solution and (iv) redundant constraint? How would you detect them while solving a problem using graphic method. **[B. Com. (H) III Sem. 2012]**