Chapter 4

Choice

Learning Objectives :

After learning this chapter you will understand :

- **Optimal Choice.**
  - ✓ **Interior Optimum,**
  - ✓ **Boundary Optimum.**
- **Tangency Condition.**
- **Optimal Choice Under Special Cases.**
  - ✓ **Perfect Substitutes,**
  - ✓ **Perfect Compliments,**
  - ✓ **Neutrals,**
  - ✓ **Bads,**
  - ✓ **Discrete Goods,**
  - ✓ **Concave Preferences,**
  - ✓ **Cobb-Douglas Preferences**
- **Implications of MRS Condition.**
- **Income Tax versus Quantity Tax.**

Basic Concepts

1. **Consumer's Equilibrium :** A consumer is said to be in equilibrium when he maximizes his utility, given his income and the market prices. In the indifference curve technique, we normally discuss the consumer's equilibrium in respect of the purchase of two goods by the consumer. The consumer is assumed to be rational in the sense that he aims at maximizing his satisfaction. Besides, the following assumptions are also made :

   (i) The consumer has a given indifference map exhibiting his scale of preferences for various combinations of two goods, say, X and Y. A single indifference curve in the map shows various combinations of these goods which yield the same utility to the consumer.

   (ii) He has a fixed amount of money to spend on the two goods. Moreover, he spends whole of the given money on two goods.

   (iii) Prices of the goods are given and are constant. Various combinations of goods X and Y which the consumer can purchase with his income are shown by the price line.

   (iv) Goods are homogeneous and divisible.

   To show which combination of two goods X and Y, the consumer will actually buy and will be in a state of equilibrium will be determined by the combination of his indifference map and price line. Consider the ***Figure 1*** in which consumer's indifference map together with the price line 'AB' is depicted.

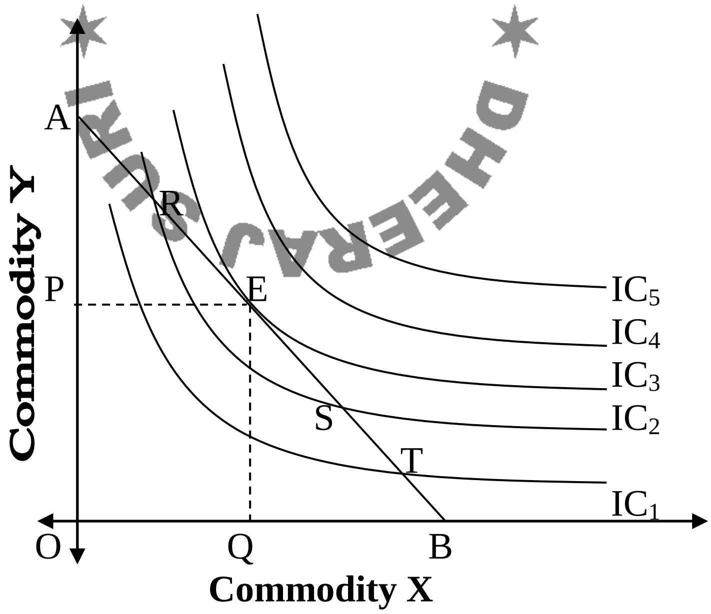

Figure 1: Optimal Choice

Commodity X is measured on the X-axis and commodity Y is measured on the Y-axis. With given money to be spent and given prices of the two goods, the consumer can buy any combination of the goods which lies on the price line AB. Any combination on the price line AB costs him the same amount of money. In order to maximize his satisfaction the consumer will try to reach the highest possible indifference curve which he can with given amount of money and given prices of the two goods. Budget constraint forces the consumer to remain on the given price line, that is, to choose any combination from among only those which lie on the given price line.

It can be seen from ***Figure 1*** that the various combinations of the two goods lying on the price line AB and which he can afford to buy do not lie on the same indifference curve, rather they lie on different curves. The consumer will choose that combination on the price line AB which lies on the highest possible indifference curve. The highest indifference curve to which the consumer can reach is the indifference curve to which price line is tangent. Any other possible combination would either lie on the lower indifference curve or would be unattainable.

In ***Figure 1***, the price line AB is tangent to indifference curve IC_3 at point E. since indifference curves are convex to the origin, all other points on the price line AB, would lie on the lower indifference curves. Take point R which also lies on the price line AB and, therefore, costs him the same as the combination E. similar is the case with other points on AB such as S, T etc. Of course, combinations lying on indifference curves IC_4 and IC_5 will give greater satisfaction than E but they are unattainable with the given amount of money and prices of the goods.

Thus, the consumer will obtain the maximum possible satisfaction and, therefore, will be in equilibrium position at point E at which the price line AB is tangent to the indifference curve IC_3. In this position, the consumer will buy OQ amount of good X and OP amount of good Y. At tangency point E, the slope of price line AB and slope of IC_3 are equal.

Slope of indifference curve = $MRS_{XY} = \frac{MU_X}{MU_Y}$,

While slope of price line AB indicates the ratio between the prices of the two goods, i.e., Slope of price line AB = $\frac{P_X}{P_Y}$.

Thus at equilibrium point E,

Slope of Indifference curve = Slope of price line AB

i.e., $\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$.

Note, that in case of perfect substitutes this tangency condition will not be valid. But, here the optimal consumption bundle will be a corner solution.

2. **The Tangency Condition :** So far we identified the two possible types of solutions we might find for the optimization problem : interior solutions and corner solutions. In general this may not tell us very much. However, this is a very useful differentiation in the case where the consumer's indifference curves are differentiable. For the purposes of this course, we can think of a differentiable function as one that is 'smooth', in the sense that it has a unique tangent (and therefore unique slope) at any point. Of all the functions that we have come across so far in the previous chapters, the only one that is not differentiable is min(x_1, x_2) at its vertices. This function does not have a unique slope at its kink.

The interesting property of interior solutions for differentiable functions is that, at any such solution, ***the slope of the indifference curve has to be the same as that of the budget line*** - in other words they are tangent. This is an incredibly useful property, and one that you can see this from ***Figure 2***. This shows two points at which the budget line is not tangent to the indifference curves - (y_1, y_2) and (z_1, z_2) and one where it is (x_1, x_2). If we look at (y_1, y_2), it is clear that this is not an optimal consumption bundle - the consumer could move onto a higher indifference curve by swapping good 1 for good 2 - *i.e.*, by moving down the budget constraint.

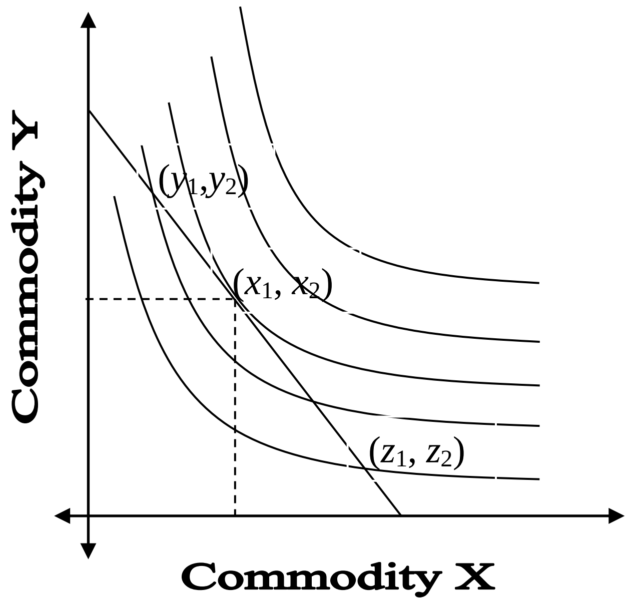

Figure 2: Optimal Choice

Similarly, at (z_1, z_2), you can move to a higher indifference curve by swapping good 1 for good 2. However, at (x_1, x_2), there is nowhere to go - either moving up or down the budget line will move the consumer to a lower indifference curve. Thus, this is (or at least could be) the optimal consumption bundle.

This tangency condition also makes sense: at the tangency point, we know that the slope of the budget line equals the slope of the indifference curve. The first of these slopes is the rate at which the consumer can trade off good 1 for good 2 in the market, while the second measures the rate at which the consumer trades off good 1 for good 2 while remaining indifferent. If there were a difference between these two ratios, then the consumer could move to a different indifference curve because the market prices would either over- or under- value good 1 relative to good 2 compared to the preferences of the consumer.

The consumer could therefore swap good 1 for good 2 (or vice versa) and so move to a higher indifference curve. Thus, it is only when the two are equal that the market 'values' the tradeoff of good 1 to good 2 at the same rate as the consumer, and so they cannot move on to a higher indifference curve.

Note **Tangency condition is necessary condition for optimal choice for well behaved preferences having interior optimum, *i.e.*, In case of well behaved preferences, if the optimal choice involves consuming some of both goods, so that it is an interior optimum, then necessarily the indifference curve will be tangent to the budget line.**

3. **Exceptions of Tangency Condition :** So we know that any interior solution has to be a tangency point. But this tangency condition doesn't hold in all the cases. If the preferences are not well behaved then at the optimum point the tangency condition

will not hold. What is always true that at the optimal point the indifference curve will not cross the budget line. The following examples illustrate the situations where at the optimum point the budget line is not tangent to the indifference curve and still indifference curve is not crossing the budget line :

(i) **Kinky Preferences :** If the shape of indifference curves is kinky, then they are not differentiable at the point where they make the kink. So, in such a case the budget line is not tangent to the indifference curve at the optimal point. As shown in ***Figure 3***, that at the optimal point E the budget line passes through the kink made by the indifference curve, but at point E the indifference curve is not differentiable. Hence at such optimal point the tangency condition isn't satisfied, and also the indifference curve is not crossing the budget line at point E.

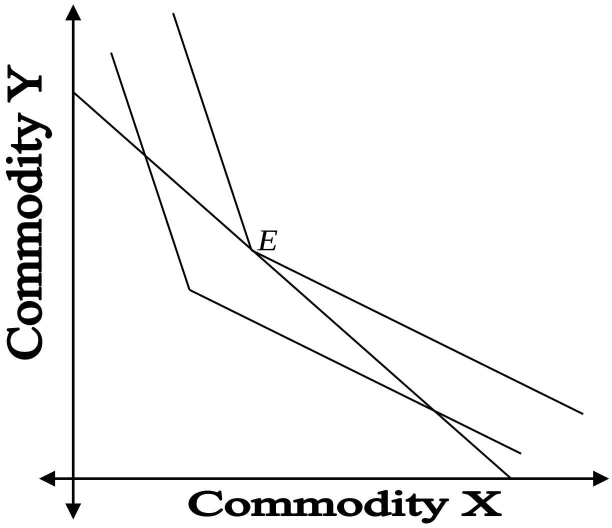

Figure 3: Kinky Preferences

(ii) **Boundary Optimum :** If the optimal point lies at either of the axes then such optimal point is called a boundary optimum. This situation occurs if the consumption of one of the goods is zero. If the consumption of good X is zero then the optimum point will lie on Y-axis, whereas if the consumption of good Y is zero then the optimum point will lie on X-axis. At the boundary optimum the slope of indifference curve and the slope of the budget line are different but the indifference curve still doesn't cross the budget line.

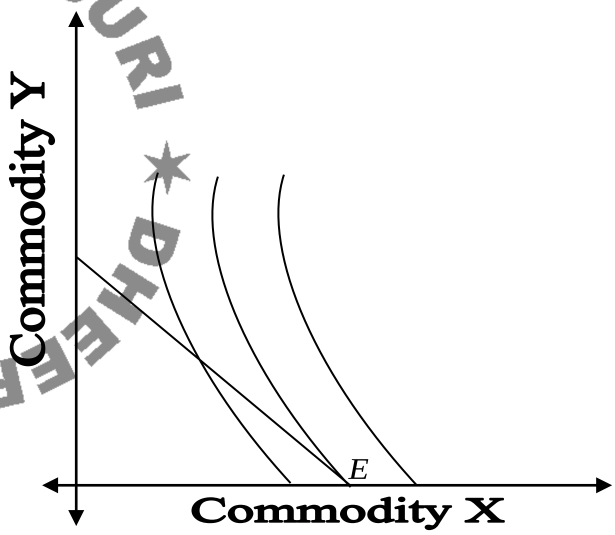

Figure 4: Boundary Optimum

As shown in **Figure 4**, that at point E the indifference curve meets the budget line, and point E lies on X-axis so it is a boundary optimum. At this optimum point the budget line is not tangent to the indifference curve. Also, the indifference curve is not crossing the budget line at such optimum point.

4. **Sufficiency of Tangency Condition :** As we know that any interior solution has to be a tangency point. Is it the case that ***any tangency point is also an interior solution? The answer is no!*** In fact, sometimes the tangency point is the ***worst*** bundle of goods that the consumer could choose while spending all their money. The following examples deals with the situation where the tangency points are not interior solutions :

(i) **Concave Preferences :** In ***Figure 5*** we look at indifference curves for concave preferences - the opposite of convex preferences (*i.e.*, people prefer 'extreme' bundles to average bundles). Here, we can certainly find a point of tangency - (x_1, x_2). However, we are now finding the worst indifference curve that is feasible from the budget line - any other point on the budget line would put the consumer on a higher indifference curve. The optimal bundle here is in fact a corner solution - (y_1, y_2). In this case the tangency point is the ***worst*** bundle of goods that the consumer could choose while spending all of her money.

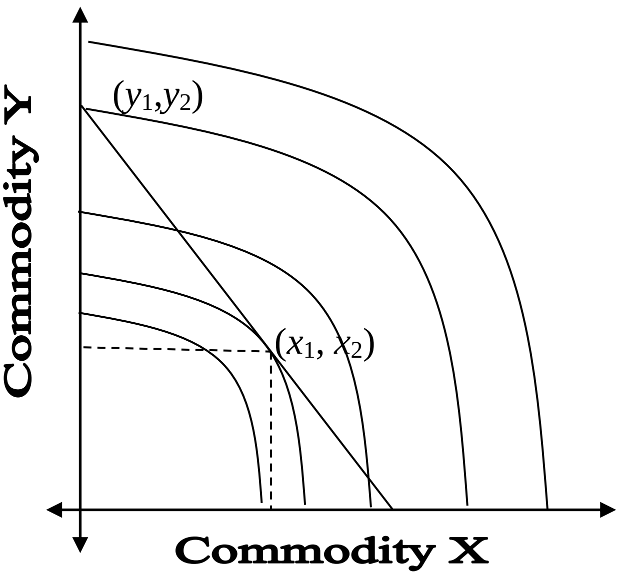

Figure 5: Concave Preferences

(iii) **Multiple Tangencies :** ***Figure 6*** shows a case where we have more than one tangent points and all of them are interior points but here all the tangent points are not optimal points. In this figure, the optimal bundles are (y_1, y_2) and (w_1, w_2) and as we can see that they are tangency points. However, the other tangency point - such as (x_1, x_2) is not optimal bundle. So, being a tangency point doesn't sufficiently make an optimal bundle.

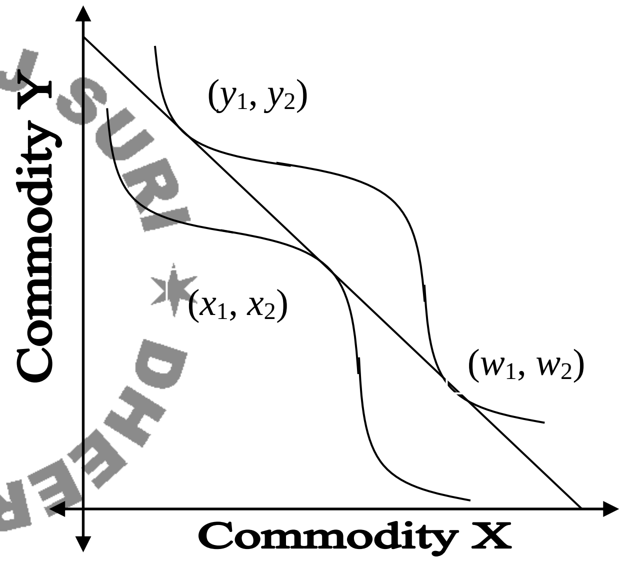

Figure 6: Multiple Tangencies

5. **Conclusion of Tangency Condition :** So, *the tangency condition is necessary for a bundle to be optimal, but not sufficient*. This is just restating that any interior optimum must be a tangency point, but not all tangency points are optimal. Thus, tangency conditions are not 'magic bullets', that give us a solution to the problem for free. However, they do suggest a procedure for finding the optimal bundle. We know that either the solution to the optimization problem will be a corner solution (*i.e.*, $x_1 = 0$ or $x_2 = 0$), or it will be a point of tangency. Thus, a sure fire (if brute force) way of finding the optimal bundle is to list all such points (i.e. all points of tangency and all corner solutions) and determine which is the best of those bundles. Unless you have a reason for a priori ruling out some possible solutions to the optimization problem, then this is the approach you should take to solving the problem.

In general there can be more than one optimal bundles that satisfy the tangency condition. However convexity implies a restriction. If the indifference curves are strictly convex, *i.e.*, they don't have flat spots, then there will be only one optimal choice on each budget line and such optimum point will be an interior optimum.

One case in which tangency conditions are both necessary and sufficient is when preferences are strictly convex - for example those preferences which are represented in **Figure 2**. An examination of this graph should tell you the following facts :

(i) There can be at most one tangency point between strictly convex, monotone preferences and the budget constraint.

(ii) Such a point must be an optimal consumption bundle.

6. **Demand Function :** The optimal choice of the consumer depends upon the price of commodities and his income. The function which relates the optimal choice of the consumer with the different values of prices and incomes is called the demand function. Generally we write the demand functions as depending on both prices and income, *i.e.*, $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$. These functions show that for each different set of prices and income, there will be a different combination of goods which show the optimal choice of the consumer.

7. **Optimal Choice Under Special Cases :** If p_1 and p_2 are prices of two commodities and x_1 and x_2 are optimal choices, M is the income of the consumer. The budget line is $p_1x_1 + p_2x_2 = M$. The optimal choices are obtained as under in some special cases :

(i) **Perfect Substitutes :** The two goods can be perfect substitutes when they can be substituted with each other on some fixed basis. The simplest case is when goods are substituted on one to one basis. For perfect substitutes the indifference curves are negatively sloped straight lines. In case of perfect substitutes the consumer will spend his entire income on the cheaper good and will not purchase the dearer good at all. For the utility function U = $x_1 + x_2$, there are three possible situations :

(a) If $p_1 < p_2$, then the budget line is flatter then the indifference curve. Here good 1 is cheaper, so the consumer will spend his entire income on good 1 and will not purchase good 2 at all. As shown in ***Figure 7(a)*** the budget line AB meets the highest possible indifference curve at point B, where consumer does not buy good 2 at all.

(b) If $p_1 > p_2$, then the budget line is steeper then the indifference curve. Here good 2 is cheaper, so the consumer will spend his entire income on good 2 and will not purchase good 1 at all. As shown in ***Figure 7(b)*** the budget line AB meets the highest possible indifference curve at point A, where consumer does not buy good 1 at all.

(c) If $p_1 = p_2$, then the slope of budget line is equal to the slope of the indifference curve and the budget line coincides with indifference curve. In this case there is a whole range of optimal choices. Any amount of goods 1 and 2 that satisfy the budget constraint is optimal solution, so the entire budget line represent the optimum solutions. In this case consumer doesn't care which one good he or she purchases.

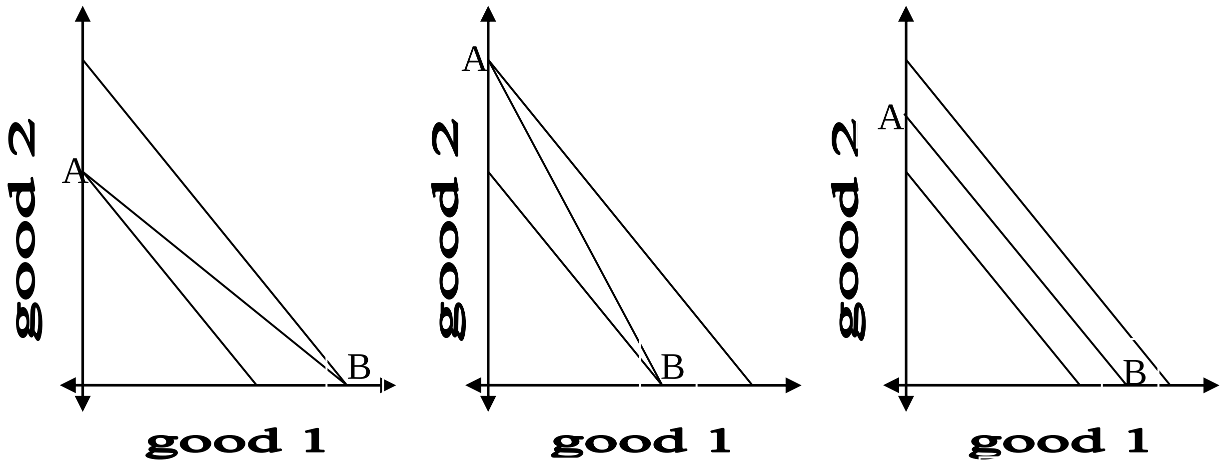

Figure 7: Perfect Substitutes

Demand Function of Perfect Substitutes : In case of perfect substitutes if β units of good 1 are substituted with α units of good 2 then the utility function is $U(x_1, x_2) = \alpha x_1 + \beta x_2$. The optimal consumption bundle is obtained as under:

$$x_1 = \begin{cases} \frac{M}{p_1} & if \frac{p_1}{p_2} < \frac{\alpha}{\beta} \\ any\ number\ between\ 0\ and\ M/p_1 & if \frac{p_1}{p_2} = \frac{\alpha}{\beta} \\ 0 & if \frac{p_1}{p_2} > \frac{\alpha}{\beta} \end{cases}$$ and

$$x_2 = \begin{cases} \frac{M}{p_2} & if \frac{p_1}{p_2} > \frac{\alpha}{\beta} \\ any\ number\ between\ 0\ and \frac{M}{p_2} & if \frac{p_1}{p_2} = \frac{\alpha}{\beta} \\ 0 & if \frac{p_1}{p_2} < \frac{\alpha}{\beta} \end{cases}$$

If goods are perfect substitutes the optimal choice will usually lie on the boundary, i.e., we will get boundary optimum.

(ii) **Perfect Compliments :** Two goods are perfect complements when they are always consumed together in some fixed ratio. The simplest case is when the goods are consumed in the ratio of 1 : 1. For perfect complements the indifference curves are 'L' shaped. On each IC the consumer has only one choice, *i.e.*, at the kink.

It means that all the optimal choices must lie on the diagonal obtained by joining these kinks. As shown in ***Figure 8***, the budget line AB, meets the highest possible IC at point E, which represents the optimal choice.

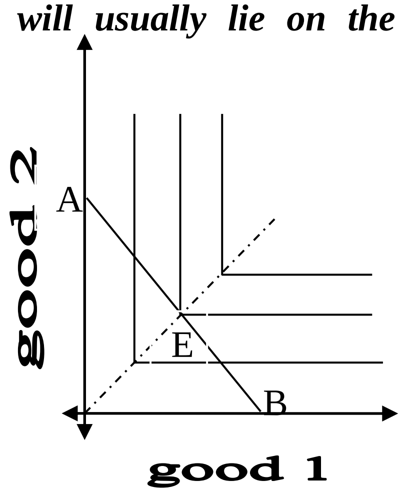

Figure 8: Perfect Complements

It should be noted that the indifference curve is not tangent to the budget line at point E.

Demand Function of Perfect Complements : In case of perfect Complements the utility function is $U(x_1, x_2) = \min(\alpha x_1, \beta x_2)$. The optimal consumption bundle is obtained as under :

$$x_1 = \frac{\beta M}{\beta p_1 + \alpha p_2} \quad \text{and} \quad x_2 = \frac{\alpha M}{\beta p_1 + \alpha p_2}$$

If the goods are perfect complements, the quantities demanded will always lie on the diagonal since the optimal choice occurs where αx_1 equals βx_2.

(iii) **Neutrals :** A good is a neuter if the consumer doesn't care it one way or the other. If one of the goods consumed is neuter then the consumer spends all of her money on the good she likes and doesn't purchase any amount of the neutral good. In the case of a neutral good the indifference curves are straight lines parallel to an axis. If good 1 is neuter, as shown in ***Figure 9(a)***, then the indifference curves are straight lines parallel to X-axis and the optimal point A lies on Y-axis, here consumer spends his entire income on good 2 and doesn't purchase good 1 at all. Whereas, If good 2 is neuter, as shown in ***Figure 9(b)***, then the indifference curves are straight lines parallel to Y-axis and the optimal point B lies on X-axis, here consumer spends his entire income on good 1 and doesn't purchase good 2 at all.

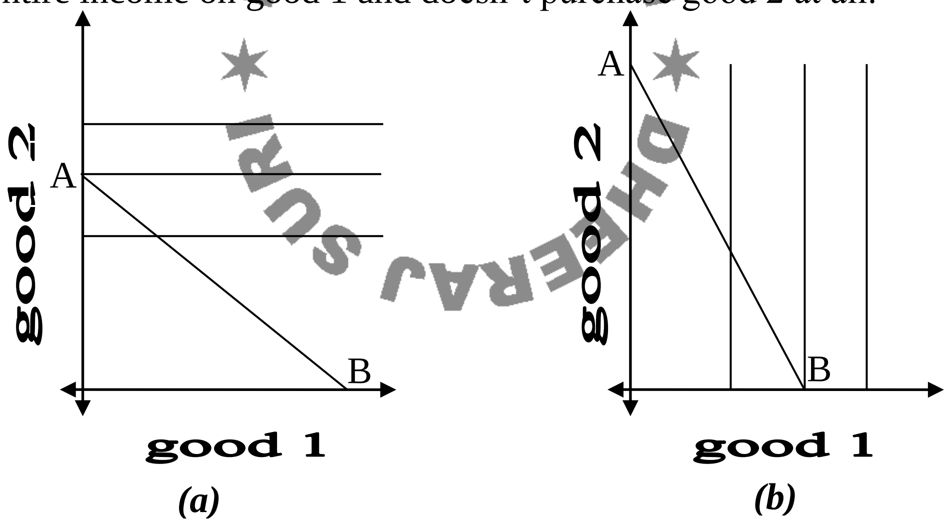

Figure 9 : Neutrals

Demand Function in Case of Neuters : If good 1 is an economic good and good 2 is a neuter, then the demand functions will be as under :

$x_1 = \frac{M}{p_1}$ and $x_2 = 0$.

Whereas, If good 2 is an economic good and good 1 is a neuter, then the demand functions will be as under :

$x_1 = 0$ and $x_2 = \frac{M}{p_2}$.

(iv) **Bads :** A commodity which the consumer does not like at all is called bad. If one of the goods consumed is bad then the consumer spends all of her money on the good she likes and doesn't purchase any amount of the bad commodity. If good 1 is bad, as shown in ***Figure 10(a)***, then the indifference curves are upward sloping with preference direction north west and the optimal point A lies on Y-axis, here consumer spends his entire income on good 2 and doesn't purchase good 1 at all. Whereas, If good 2 is bad, as shown in ***Figure 10(b)***, then again the indifference curves are upward sloping but having preference direction of south east and the optimal point B lies on X-axis, here consumer spends his entire income on good 1 and doesn't purchase good 2 at all.

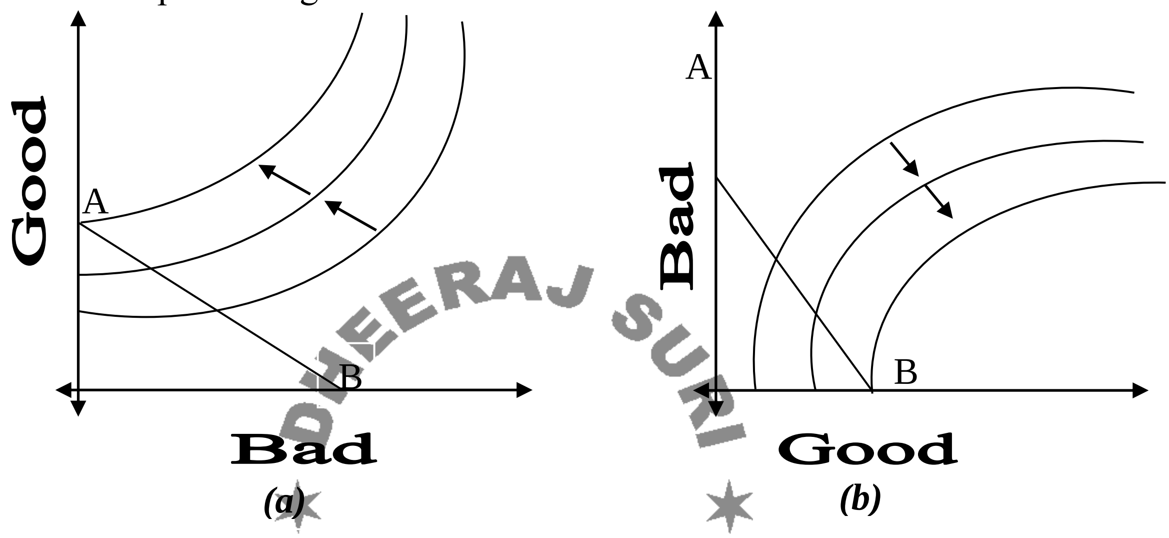

Figure 10 : Bads

Demand Function in Case of Bads : If good 1 is an economic good and good 2 is a bad, then the demand functions will be as under :

$x_1 = \frac{M}{p_1}$ and $x_2 = 0$.

Whereas, If good 2 is an economic good and good 1 is a bad, then the demand functions will be as under :

$x_1 = 0$ and $x_2 = \frac{M}{p_2}$.

(v) **Discrete Goods :** Suppose that good 1 is a discrete good that is available only in integer units, while good 2 is money to be spent on everything else. If the consumer chooses 1, 2, 3, …., units of good 1, she will implicitly choose the consumption bundles (1, m – p_1), (2, m – 2p_1), (3,m – 3p_1), and so on. We can simply compare the utility of each of these bundles to see which has the highest utility.

If the consumer finds good to be very costly then she will not buy good 1 at all. As price of good 1 decreases she may decide to consume 1, 2 or more units of good 1. In case of discrete goods there may be more than one optimal points, this may happen if some portion of the budget line coincides with the indifference curve.

(vi) **Concave Preferences :** Repulsive goods have concave preferences. Those goods which the consumer doesn't like to consume together are repulsive goods. Here the consumer has to choose only one good and he chooses the cheaper one. As shown in ***Figure 5*** above that the concave preferences have boundary optimum and not the interior tangency point, because extremes are preferred to averages, So, the boundary point would lie on a higher indifference curve. Thus, in this case we always have boundary optimum and the consumer will consume either of the two commodities.

(vii) **Cobb-Douglas Preferences :** Cobb-Douglas preferences are well behaved preferences, *i.e.*, they have downward sloping and convex indifference curves. So, these preferences will certainly have internal optimum where some of both goods are being consumed. Suppose that the utility function is of the Cobb-Douglas form $U(x_1, x_2) = x_1^c x_2^d$. The optimal choices in this case are:

$$x_1 = \frac{c}{c+d}.\frac{M}{p_1} \quad \text{and} \quad x_2 = \frac{d}{c+d}.\frac{M}{p_2}$$

Cobb-Douglas preferences have a convenient property we can easily determine the fraction of income spent on either of the goods. The fraction of income spent on good 1 is $\frac{c}{c+d}$ whereas the fraction of income spent on good 2 is $\frac{d}{c+d}$. Thus the Cobb-Douglas consumer always spends a fixed fraction of her income on each good. the size of the fraction is determined by the exponent in the Cobb-Douglas function.

8. **Implications of the MRS Condition :** Suppose that there are two goods in the market and market prices are same for everyone then all the consumers will have the same MRS at their optimal points. In well-organized markets, it is typical that everyone faces roughly the same prices for goods, *i.e.*, The market is offering everyone the same rate of exchange for the two goods, and everyone is adjusting their consumption of the goods until their own "internal" marginal valuation of the two goods equals the market's "external" valuation of the two goods.

Now the interesting thing about this statement is that it is independent of income and tastes. People may value their total consumption of the two goods very differently. Some people may be consuming a lot of good 1 and a little of good 2, and some may be doing the reverse. Some wealthy people may be consuming a lot of both goods while other people may be consuming just a little of each good. But everyone who is consuming the two goods must have the same marginal rate of substitution.

Also, as we know prices measure the rate at which people are just willing to substitute one good for another, they can be used to value policy proposals that involve making changes in consumption. The fact that prices are not arbitrary numbers but reflect how people value things on the margin is one of the most fundamental and important ideas in economics.

9. **Income Tax versus Quantity Tax :** An income tax is charged on the income of the consumer and it is independent of the number of units of either good being consumed by the consumer. Income tax causes a fall in income so the budget line would shift leftwards but remaining parallel to the original budget line.

Whereas, a quantity tax is charged on the number of units of a good being consumed by the consumer. A quantity tax raises the price of the commodity on which it is being charged so it rotates the budget line.

It can be shown using indifference curve analysis that raising revenue through income tax is better for consumer's welfare than raising it through the quantity tax given the assumptions that :

(i) The income of the consumer is m and the prices of two goods x_1 and x_2 are p_1 and p_2 respectively.

(ii) The consumer is rational and tries to maximize his utility subject to the budget constraint.

(iii) The revenue collected from both type of taxes is the same for the government.

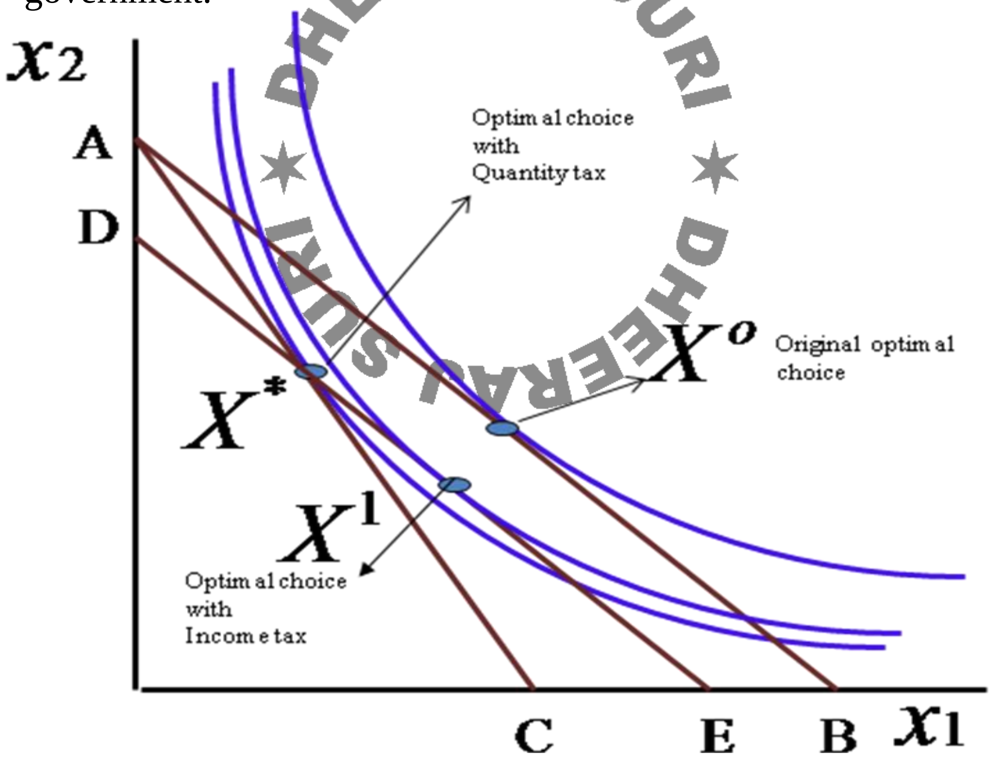

Figure 11 : Income Tax versus Quantity Tax

We begin our analysis by first considering the quantity tax. As shown in **Figure 11**, the original budget line,

$$p_1x_1 + p_2x_2 = m$$

is represented by AB and is tangent to an indifference curve at point $\boldsymbol{X^0}$. When the government imposes a quantity tax at the rate of Rs. t per unit on good 1 then the new budget line becomes

$$(p_1 + t)x_1 + p_2x_2 = m$$

This new budget line is represented by AC and it is tangent to an IC at point $\boldsymbol{X}^*$ and here optimal choice is (x_1^*, x_2^*). At this stage we do not know that whether the quantity tax on good 1 increases or decreases its consumption, but in either case the optimum bundle (x_1^*, x_2^*) must satisfy the budget constraint, i.e.,

$$(p_1 + t)x_1^* + p_2x_2^* = m$$

So, the revenue raised by the government through quantity tax is $R^* = t.x_1^*$

Now, if government charges an income tax instead of quantity tax and the magnitude of income tax is $R^* = t.x_1^*$, then it will reduce the income of the consumer and the new budget line will become

$$p_1x_1 + p_2x_2 = m - R^*$$

It is represented by line DE in the above figure, DE is parallel to original budget line AB and it passes through the optimum point obtained with quantity tax. DE is tangent to an indifference curve at point $\boldsymbol{X^1}$. the IC to which DE is tangent is higher than the IC to which AC is tangent, this shows that with same amount of tax revenue to the government the consumer's welfare is increased with income tax.

Theory Questions

Q1. Suppose that a consumer spends his entire income on the purchase of two goods. Explain why a consumer will choose a market basket so that the MRS between two goods is equal to their price ratio.

Q2. If you are convinced that you are really choosing the best bundle of goods that you can buy with your income, what is an easy way to determine your MRS for any pair of goods? **[Eco. (H) 1989]**

Q3. If a consumer maximize utility subject to his budget constraint. How can you estimate his MRS?

Q4. Marginal rate of substitution between two goods is equal to the ratio of their prices. What does it imply? Explain graphically. **[Eco. (H) 1996]**

Q5. Suppose MRS between goods X and Y increases (in absolute value) as good X is substituted for good Y. What would the tangency between budget line and indifference curve indicate? What would be the most preferred consumption choice?

Q6. A consumer suddenly realizes that MU_x/p_x is less than MU_y/p_y, with the current commodity bundle. Is he maximizing his utility? If not which good should be consumed more of, to improve utility without increasing the expenditure? What happens to these ratios as he begins adjusting his consumption ? why ? **[BBE I Sem. 2011]**

Q7. Explain corner solution in consumer equilibrium for different shapes of indifference curves. **[BBE I Sem. 2010]**

Q8. Assume a two commodity world. Is tangency between the indifference curve and the budget line a necessary condition for a bundle to be optimal? Substantiate your answer through the following situations : **[Eco. (H) 1999]**

(i) Kinked indifference curves.

(ii) Strictly convex preferences leading to a corner optimum.

Q9. Mr. X likes shoes and leather purses. If he doubles the number of pairs of shoes, he need only decrease his consumption of purses by half to be equally well off. Draw an indifference curve to reflect the above preference relation. **[Eco. (H) 2003]**
[**Hint :** Rectangular hyperbola shaped IC's]

Q10. If a consumer has a utility function u(x_1, x_2) = $x_1.x_2^4$, what fraction of her income will she spend on good 2?

Ans. 4/5

Q11. If a consumer has a utility function u(x_1, x_2) = $x_1^2.x_2^4$, what fraction of her income will she spend on good 1?

Ans. 1/3

Q12. Illustrate using indifference curves how consumers can be made worse off when a product that they consume is rationed.

Q13. Suppose that indifference curves are described by straight lines with a slope of –b. Given arbitrary prices and money income p_1, p_2 and m, what will the consumer's optimal choices look like?

Ans.
$$x_1 = \frac{m}{p_1},\ x_2 = 0 \qquad if\ \frac{p_1}{p_2} < b$$
$$x_1 = 0,\ x_2 = \frac{m}{p_1} \qquad if\ \frac{p_1}{p_2} > b$$

Any combination of x_1 & x_2 on the budget line if $\frac{p_1}{p_2} = b$

Q14. Draw indifference preference map between two goods that are perfect substitutes. What can you say about consumer's equilibrium? **[Eco. (H) 1992]**

Q15. The case of perfect substitutes arises when all that matters to the consumer is the sum of the products – *e.g.* red shirts and green shirts for a color-blind consumer. In this case, $u(x, y) = x + y$. Graph the indifference curves for perfect substitutes. Show that the consumer maximizes utility by spending their entire income on whichever product is cheaper.

Q16. The slope of indifference curve and the budget line are same at all levels of consumption. What can you say about the equilibrium of the consumer? **[Eco. (H) 1996]**

Q17. Coke and Pepsi are perfect substitutes for Arjun and the slope of his indifference curve is 1. What should be the price ratio if the optimal bundle consists of positive amount of both Coke and Pepsi? **[Eco. (H) 1998]**

Q18. A consumer has a utility function with the form : **[Eco. (H) 2003]**

$$u(x_1, x_2) = (x_1)^a + (x_2)^b$$

Where it is assumed that $a \neq 0$ and $b \neq 0$. For each of the following additional properties, what additional restrictions are required on the two parameters a and b ?

(i) Preferences are quasi-linear.

(ii) Two goods are perfect substitutes.

Ans. (i) $a > 1$ and $b = 1$, (ii) $a = 1$ and $b = 1$.

Q19. Draw indifference curves for the following two pairs of goods : Tea and Sugar, Tea and Coffee. In which case the consumer responds more to change in the relative prices? Explain. **[Eco. (H) 2004]**

Q20. If a consumer's indifference curves are concave, will he consume both the goods together? **[Eco. (H) 1997]**

Q21. If a consumer's indifference curves are concave, will he consume both the goods in equilibrium? Why or why not?

[**Hint :** No, he would buy only one commodity.]

Q22. Do you agree with the assertion that a lump sum tax is superior to a quantity tax that gives the same revenue to the government?

Q23. Show using indifference curve analysis that 'lump sum' subsidy makes the consumer better off compared to an 'excise subsidy' which costs the government the same amount **[Eco. (H) 2007]**

Q24. For what kind of preferences will the consumer be just as well-off facing a quantity tax as an income tax?

[For kinked preferences, such as perfect complements, where the change in price doesn't induce any change in demand.]

Numerical Problems

Q1. An individual's marginal utilities for commodity X and Y are given as :

$MU_x = 40 - 5x$, $MU_y = 20 - 3y$

What is the marginal rate of substitution if the consumer is consuming 3 units of X and 5 units of Y? If $P_x = 5$ and $P_y = 1$, will the consumer be in equilibrium?

Ans. MRS = 5, Yes

Q2. Assume that a consumer who consumes only two goods A and B and has his MRS given as :

$$MRS = MU_A/MU_B = B/A$$

If his money income is Rs. 100, price of A is Rs. 5 and price of B is Rs. 10, what quantities of the two goods will he consume? **[Eco. (H) 2005]**

Q3. If a consumer has the utility function $U = x_1 + x_2$ and $P_2 = 1$. What is his reservation price for good x_1 as a function of x_2 and M?

Ans. $P_1 = M - x_2$.

Q4. Suppose you have Rs. 21 per week to allocate between Hamburger and Puri Chole. The Hamburger costs Rs. 3 and Puri Chole Rs. 1.50. Last month you chose to buy 6 Puri Choles and 4 Hamburgers for lunch. Now, you are aware of MRS concept, you have determined that your MRS based on last month's purchase is 3 Puri

Choles for one Hamburger. Determine whether last month's choice is optimal and if not, how it should be changed. **[Eco. (H) 1989]**

Q5. Vinita likes both tennis rackets and tennis shoes and would like to consume both. At the moment, she has many of both and her marginal rate of substitution (MRS) of rackets for shoes is 3. Unused rackets and shoes may be returned to the store for a refund. The current price for a racket is rupees 200 and the price for a pair of shoes is rupees 100. Is her present consumption bundle an optimum? Suggest a way for Vinita to make herself better off. **[Eco. (H) III Sem. 2016]**

Find the optimal consumption bundles by all three methods (Q5 to 17)

Q6. An agent's utility function is written as $U = XY$, and his budget constraint is $5X + 2Y = 100$. What are the optimal amounts of X and Y?

Ans. $X = 10$; $Y = 25$

Q7. An agent's utility function is written as $U = X^2Y^2$, and his budget constraint is $X + Y = 100$. What are the optimal amounts of X and Y?

Ans. $X = 50$; $Y = 50$

Q8. An agent's utility function is written as $U = X^3Y$, and his budget constraint is $X + 3Y = 50$. What are the optimal amounts of X and Y?

Ans. $X = 75/2$; $Y = 25/6$

Q9. An agent's utility function is written as $U = 5XY$, and his budget constraint is $X + 2Y = 200$. What are the optimal amounts of X and Y?

Ans. $X = 100$; $Y = 50$

Q10. An agent's utility function is written as $U = 4X^2Y$, and his budget constraint is $2X + 3Y = 60$. What are the optimal amounts of X and Y?

Ans. $X = 20$; $Y = 20/3$

Q11. An agent's utility function is written as $U = 6X^3Y^3$, and his budget constraint is $3X + 6Y = 120$. What are the optimal amounts of X and Y?

Ans. $X = 20$; $Y = 10$

Q12. An agent's utility function is written as $U = 8XY^2$, and his budget constraint is $X + Y = 45$. What are the optimal amounts of X and Y?

Ans. $X = 15$; $Y = 30$

Q13. An agent's utility function is written as $U = 10X^2Y^2$, and his budget constraint is $2X + 2Y = 300$. What are the optimal amounts of X and Y?

Ans. $X = 75$; $Y = 75$

Q14. An agent's utility function is written as $U = X^4Y^2$, and his budget constraint is $8X + Y = 48$. What are the optimal amounts of X and Y?

Ans. $X = 4$; $Y = 16$

Q15. An agent's utility function is written as $U = 12X^2Y^2$, and his budget constraint is $X + 3Y = 120$. What are the optimal amounts of X and Y?

Ans. $X = 60$; $Y = 20$

Q16. An agent's utility function is written as $U = 3X^3Y$, and his budget constraint is $X + 2Y = 400$. What are the optimal amounts of X and Y?

Ans. $X = 300$; $Y = 50$

Q17. An agent's utility function is written as $U = 4X^4Y^4$, and his budget constraint is

$4X + Y = 88$. What are the optimal amounts of X and Y ?

Ans. $X = 11$; $Y = 44$

Q18. Given the utility function $U = (x + 2)(y + 1)$, and the budget constraint $2x + 5y = 51$ find the optimum levels of x and y purchased by the consumer.

Ans. Max. U = 90 at $x = 13$, $y = 5$

Q19. Given the utility function $U = (x + 2)(y + 1)$, where x represents consumption of good 1 and y represents consumption of good 2. The price of each good is Re. 1 and the income of the consumer is Rs. 11.

(i) Write the equation of an indifference curve which passes through the point (2, 8). Also sketch this indifference curve on the graph.

(ii) Determine the MRS.

(iii) Write down the equation of budget line and also find its slope.

(iv) Can the consumer achieve the utility of 11 with this budget.

(v) Determine the optimum consumption bundle of the consumer.

Ans. **(i)** $y = \frac{34 - x}{x + 2}$, **(ii)** MRS = $-\frac{y+1}{x+2}$, **(iii)** $x + y = 11$, **(iv)** Yes,

(v) $x = 5, y = 6$.

Q20. If $u = \sqrt{xy}$ is the utility function, $P_x = P_y$ = Rs. 10 and Income = Rs. 100, find the demands that maximize utility. Check the second order condition.

Ans. Max. U = 5 at $x = 5$, $y = 5$

Q21. A consumer's utility function is given as :

$u = \sqrt{x_1 x_2}$

where x_1, x_2 denote the quantities of two products consumed by the consumer and the prices per unit of the goods are Rs. 20 and Rs. 10 respectively. Determine the optimum level of commodities to maximize his utility and spend his total income of Rs. 640 on the two goods.

Ans. Max. U = 22.63 at $x_1 = 16$, $x_2 = 32$

Q22(i) Anjali, a sports woman, enjoys playing two games X and Y each week and derives utility according to the utility function $U = \sqrt{XY}$. If she has Rs.24 a week to spend on the two games and the price of playing X and Y is Rs.4, how will she maximize utility while playing the two games?

(ii) Anjali is also a businesswoman with a very busy schedule. She has only 16 hours time available to devote to these activities each week. If time taken for game X is 4 hours and that for game Y is 2 hours, how many hours would she pursue her games under the circumstances of part (i)

Q23. Suppose $U(x, y) = x^{\alpha} + y^{\alpha}$ for $\alpha < 1$. Show that :

$$x = \frac{M}{p_x\left(1 + \left(\frac{P_y}{P_x}\right)^{\alpha}\right)} \quad \text{and} \quad y = \frac{M}{p_y\left(1 + \left(\frac{P_x}{P_y}\right)^{\alpha}\right)}.$$

Q24. A consumer spends an amount m to buy x units of one good at the price 6/unit and y units of a different good at price 10/unit. m is positive. The consumer utility function is

$U(x, y) = xy + y^2 + 2x + 2y$

Find the optimal quantities of x^* and y^* as function of m. what are the solutions for x^* and y^* if $m \leq 8$?

Ans. $x = \frac{40 - m}{24}, y = \frac{m - 8}{8}$

Q25. A consumer's utility is a function of two goods X and Y and is given by :

U = 100logX + 50logY

If the consumer's budget is Rs. 10 and the price of X is Rs. 3 and the price of Y is Rs. 1, find the quantities of X and Y that the consumer should purchase to maximize utility. What is his marginal utility of Money?

Ans. $x = 20/9$, $y = 10/3$ & Marginal utility of money = 15

Q26. A consumer has utility function $U(x, y) = (x - a)^{\alpha}(y - b)^{\beta}$, where x and y are the quantities of the two goods that he can consume. He buys the goods at fixed prices p_x and p_y, subject to a ceiling M on total expenditure. If he maximizes U, obtain the demand functions for each good in terms of income M and prices p_x and p_y. show that the total expenditure on each good is a linear function of M.

Ans. $x = \frac{\frac{\alpha}{\beta}(M - bp_y) + ap_x}{p_x\left(\frac{\alpha}{\beta} + 1\right)}, \; y = \frac{(M - ap_x) + \frac{\alpha}{\beta}bp_y}{p_y\left(\frac{\alpha}{\beta} + 1\right)}$]

Q27. Show that two utility maximizing consumers with utility functions :

$U(x_1, x_2) = x_1^{1/2}x_2^{1/2}$ and $V(x_1, x_2) = x_1^2 x_2^2$

respectively have same demand functions

Q28. A consumer has the following utility function ;

$U(x_1, x_2) = (x_1 + 2)^{1/2}(x_2)^{1/2}, x_1, x_2 \geq 0.$

Find the optimal consumption of x_1 and x_2 given prices P_1 = Rs. 6, p_2 = Re. 1 and income = Rs. 10. **[Eco. (H) 2011]**

Q29. While you are at college hostel, your parents give you 300 rupees as pocket money. You spend the entire amount on videogames (v) and comic books (c). Videogames cost Rs. 2 per unit and comic books cost Rs. 10 and your utility function is given by $U = c^2v$. **[DSE MA Ent. Eco. 1999]**

(a) Calculate your marginal rate of substitution.

(b) Find the amounts of videogames and comic books consumed in month. Sketch this in a diagram.

(c) Suppose your father restricts you to play at most 30 videogames in a month. How does your optimum consumption change? Are you better off or worse off than before?

Q30. SV College has Rs. 60,000 to spend on computers (C) and other stuff (X). The UGC wants to encourage computer literacy in colleges and the following two plans were proposed

(a) Give a grant of Rs. 10,000 to each college, that the college is free to use as it wishes.

(b) Give a matching grant, for every rupee spent on computers, the UGC gives the college Re. 0.50.

(i) Write the budget equation and draw the budget line in each case.

(ii) If SV College has preferences that can be represented by the utility function $U(C, X) = C.X$, what will be the amount spent on computers under each plan.

Ans. (i) C + X = 70000 and 0.5C + X = 60,000

Q31. Suppose Lila's income is Rs. 8,000 per month. She finds that her monthly expenditure on food is half her income and the rest is spent on all other goods. Food costs Rs. 1 per unit. She now becomes eligible for the food stamp program, wherein she is allowed to purchase food stamps at Rs. 1 per food stamp and each food stamp can be used to purchase two units of food at retail outlets. However she cannot spend more than Rs. 1,000 on food stamps.

(a) Draw a diagram to clearly depict the change in her budget set.

(b) If Lila has homothetic preferences, find her equilibrium food consumption if she joins the food stamp program. **[Eco. (H) III Sem. 2014]**

Q32. If the utility function is $u(x, y) = 4x$, where x and y are the amounts of two good consumed, then what would be the shape of indifference curves? Find the optimal bundle if income is Rs. 200, price of x is Rs. 10 and price of y is Rs. 20.

[Eco. (H) III Sem. 2014 (ER)]

Q33. For a consumer, the utility function for two goods x and y is given by $U(x, y) = 2x + 2y + (xy)^{1/2}$. He earns an income of Rupees 1000 while $P_x = 50$ and $P_y = 50$. Find his optimal consumption choice. **[Eco. (H) III Sem. 2016]**

Quasi-linear Preferences

Q34. An individual purchases quantities x_1 and x_2 of two goods whose prices are p_1 and p_2 respectively. His utility function is :

$U(x_1, x_2) = x_1 + \log x_2$.

Assuming his income is M, find the optimal quantities x_1 and x_2. Also find the marginal utility of income.

Ans. $x_1 = \dfrac{m}{p_1} - 1,\; x_2 = \dfrac{p_1}{p_2}$

Q35. An individual purchases quantities x_1 and x_2 of two goods whose prices are $p_1 = 1$ and $p_2 = 2$ respectively. His utility function is :

$$U(x_1, x_2) = 4\sqrt{x_1} + x_2$$

(i) Write the equation of an indifference curve which passes through the point (25, 0). Also sketch this indifference curve on the graph.

(ii) Determine the MRS.

(iii) if the income of the consumer is Rs. 24, write down the equation of budget line and also find its slope.

(iv) Determine the optimum consumption bundle of the consumer.

(v) If the income of the consumer increases to Rs. 34, now find his optimum consumption bundle.

(vi) If the income of the consumer falls to Rs. 9. Draw his budget line and sketch the indifference curve which passes through the point (9, 0) and find the optimum consumption bundle. Is budget line tangent to the IC at optimum point, if not then which of them is more steeper.

Ans. (i) $x_2 = 20 - 4\sqrt{x_1}$, **(ii)** MRS = $-\frac{2}{\sqrt{x_1}}$, **(iii)** $x_1 + 2x_2 = 24$, slope = -½,

(iv) (16, 4), **(v)** (16, 9), **(vi)** (9, 0), No, indifference curve.

Q36. Suppose a consumer consumes two goods X and Y and his preferences are described by the utility function $u(x, y) = \ln x + y$. The consumer's income is Rs. 120 per month and the price of y is Rs. 12 per unit.

(a) Find his optimal consumption bundle when the price of x is Rs. 3 per unit.

(b) Find his optimal consumption bundle if the price of x increases to Rs. 6 per unit. **[Eco. (H) III Sem. 2015]**

Concave Preferences

Q37. Ajay likes oats (x) and fruit juice (y) and has concave preferences between them. Price per kilogram of oats is P_x and price per litre of fruit juice is P_y. His monthly budget for the two commodities is M. **[Eco. (H) III Sem. 2012]**

(i) Draw his indifference map and comment on the behavior of the marginal rate of substitution between oats and fruit juice (MRS_{xy}), as he increase his consumption of oats. Indicate the possible optimum choices for Ajay in a representative diagram.

(ii) Specifically, if Ajay's utility function is $U = f(x, y) = x^2 + xy + y^2$, Px = Rs. 100, Py = Rs. 80 and M = Rs. 1,000, find his optimum consumption choice.

Kinked Indifference Curves

Q38. For the utility function $U = x^{1/2}y^{1/2}$, income of the consumer M = 500, prices of the commodities $p_y = 10 \ \forall\ y$ and $p_x = \begin{cases} 2 & \forall\ x \le 50 \\ 5 & \forall\ x > 50 \end{cases}$. Find the optimum point.

Q39. Consider a two commodity world : Sugar and Kerosene. Kerosene is sold through a fair price shop only, at Rs. 10 per litre; the consumer can buy any quantity she desires. Sugar sells at Rs. 10/kg in the fair price shop and at Rs. 15/kg in the open market, but the consumer can buy up to 6 kg of Sugar from the fair price shop. Suppose the consumer's income is Rs. 150. **[DSE MA Ent. Eco. 2001]**

(a) Draw her budget line.

(b) We know the consumer has convex preferences and that she consumes 8 kg of Sugar in equilibrium. Find out her optimum consumption of Kerosene.

Perfect Complements

Q40. A consumer's utility function is given by $U(x, y) = \min\{2x, y\}$. Suppose that the price of good X is Rs. 1 per unit, price of good Y is Rs. 0.75 per unit and income is Rs. 20. How many units of X and Y will the consumer demand in this situation?

Q41. Use separate graphs to draw indifference curves for each of the following utility functions :

(i) $U(x, y) = \min(2x + y, 2y + x)$

(ii) $U(x, y) = \max(2x + y, 2y + x)$

Find the equilibrium bundles for each kind of preferences given $P_x = P_y = 1$ and $M = 10$.

Q42. Ram consumes two commodities X and Y and his utility function is $\text{Min}(x + 2y, y + 2x)$. He chooses to buy 8 units of X and 16 units of Y. the price of commodity Y is Rs. 0.50. What is the price of commodity X and income of the consumer?

Q43. Anita spends all her pocket money on chocolates and ice cream. The money spent on chocolates is x and money spent on ice creams is y. Her utility function is $U(x, y) = \min\{4x, 2x + y\}$. Anita consumes 15 chocolates and 10 ice creams. The price of a chocolate is Rs. 10. Find the price of an ice cream and her pocket money.

Ans. Price = Rs. 30, Pocket money = Rs. 450.

Q44. Reena has the utility function $U(x, y) = \min\{x, y^2\}$

(i) Write an equation for the line on which all the optimum solutions lie.

(ii) Determine her utility if her consumption bundle is (1, 1). Also draw the indifference curve which passes through (1, 1).

(iii) Determine her utility if her consumption bundle is (4, 2). Also draw the indifference curve which passes through (4, 2).

(iv) Determine her utility if her consumption bundle is (16, 5). Also draw the indifference curve which passes through (16, 5).

(v) If $p_x = 1$, $p_y = 2$ and $m = 2$, determine the optimum bundle.

(vi) If $p_x = 10$, $p_y = 15$ and Reena demands 100 units of x, find her income.

Ans. **(i)** $x = y^2$, **(ii)** 1, **(iii)** 4, **(iv)** 16, **(v)** (4, 2), **(vi)** 1150

Q45. A teacher conducts two mid-term exams and uses the minimum of the two scores to determine the course grade for the student. A student Ashish wants to maximize his grade in the course. x_1 represents score of a student in the first mid-term and x_2 represents score in the second mid-term exam. He has a total of 1200 minutes to study for these exams. If he doesn't study at all he will get zero in both the exams, whereas every 10 minutes spent on first test increases the score in first test by one point and every 20 minutes spent on second test increases the score in second test by one point.

(a) Write the utility function of Ashish.

(b) Write an equation for the line on which all the optimum solutions lie.

(c) Write Ashish's budget line.

(d) Determine how much time he should spend on first test and how much on second and what is his best score.

Ans. **(a)** U = min(x_1, x_2), **(b)** $x_1 = x_2$, **(c)** $10x_1 + 20x_2 = 1200$,
(d) 400 minutes, 800 minutes and 40 marks (40, 40).

Q46. A teacher conducts two mid-term exams and uses the maximum of the two scores to determine the course grade for the student. A student Ashish wants to maximize his grade in the course. x_1 represents score of a student in the first mid-term and x_2 represents score in the second mid-term exam. He has a total of 400 minutes to study for these exams. If he doesn't study at all he will get zero in both the exams, whereas if he spends m_1 minutes for first test then his score in first test will be m_1 / 5 points and if he spends m_2 minutes for second test then his score in second test will be m_2 / 10 points.

(a) Write the utility function of Ashish.
(b) Write an equation for the line on which all the optimum solutions lie.
(c) Write Ashish's budget line.
(d) Determine how much time he should spend on first test and how much on second and what is his best score.

Ans. **(a)** U = max(x_1, x_2), **(b)** $x_1 = x_2$, **(c)** $5x_1 + 10x_2 = 400$,
(d) 400 minutes, 0 minutes and 80 marks (80, 0).

Q47. Mrs. Pathak is very particular about her consumption of tea. She always takes 50 grams of sugar with 20 grams of ground tea. She has allocated Rs. 55 for her spending on tea and sugar per month. (Assume that she doesn't offer tea to her guests or anybody else and she doesn't consume sugar for any other purpose). Sugar and tea are sold at 2 paisa per 10 grams and 50 paisa per 10 grams respectively. Determine how much of tea and sugar she demands per month.

Q48. Picabo, an aggressive skier, spends her entire income on skis and bindings. (Binding are the mechanism by which skiers attach their boots to the skis.)

(i) If Picabo wears out one pair of bindings for every one pair of skis, graph her indifference curves for skis and bindings, illustrating bindings on the horizontal axis and skis on the vertical axis.
(ii) If Picabo wears out two pairs of bindings for every one pair of skis, graph her indifference curves for skis and bindings, illustrating bindings on the horizontal axis and skis on the vertical axis.

Now assume that Picabo has $5,760 in income to spend on binding and skis each year. Skis cost Rs.480 per pair, and bindings cost Rs.240 per pair.

(a) Graph Picabo's optimal consumption bundle for skis and bindings under the assumptions in part (i).
(b) Graph Picabo's optimal consumption bundle for skis and bindings under the assumptions in part (ii).

Ans. **(a)** S = 8, B = 8, **(b)** S = 6, B = 12

Q49. An individual always consumes one cheese slice with two bread slices and the prices of a cheese slice and a bread slice are Rs. 3 and Rs. 1 respectively.

(a) How much of the two commodities will he consume under alternate incomes of Rs. 30 and Rs. 60?
(b) State whether the utility function is homothetic?

[Eco. (H) III Sem. 2014(ER)]

Perfect Substitutes

Q50. Vivek spends all his pocket money on Pizzas (P) and Burgers (B). His utility function is given by U = 3P + B. A burger costs Rs. 20 and a pizza costs Rs. 30, and his pocket money is Rs. 300. What is vivek's optimum consumption bundle?
[DSE MA Ent. Eco. 2000]

Q51. Suppose that a consumer always consumes 2 spoons of sugar with each cup of coffee. If the price of sugar is p_1 per spoonful and the price of coffee is p_2 per cup and the consumer has m dollars to spend on coffee and sugar, how much will he or she want to purchase?

Ans. Let number of cups of coffee is x and number of teaspoons of sugar is $2x$, then

$$x = \frac{m}{2p_1 + p_2}.$$

Q52. Sara a consumer who views Chocolate and Vanilla Ice cream as perfect substitutes. She likes both and is always willing to trade one scoop of chocolate ice cream for two scoops of vanilla ice cream. If the price of a scoop of a chocolate ice cream is three times the price of vanilla ice cream, will Sara buy both types of ice cream? What happen if the price of chocolate ice cream is twice the price of vanilla ice cream? Draw diagrams to illustrate your answer. **[Eco. (H) 2003]**

Q53. Paula, a former actress, spends all her income attending plays and movies. She likes plays exactly three times as much as she likes movies.

(i) Graph Paula's indifference curves, illustrating plays on the horizontal axis and movies on the vertical axis.

(ii) Paula earns \$120 per week. If tickets to plays cost \$12 each and tickets to movies cost \$5 each, graph her optimal consumption bundle, illustrating plays on the horizontal axis and movies on the vertical axis.

Ans. P = 10, M = 0

Q54. Mala is very flexible. She consumes x and y. she says, "Give me x or give me y, I don't care, I can't tell the difference between them." She is currently endowed with 8 units of x and 17 units of y. The price of x is 3 times the price of y. Mala can trade x and y at the going prices but has no other source of income. How many units of y will Mala consume? **[Eco. (H) III Sem. 2016]**

Q55. A consumer always consumes one unit of good X with 2 units of good Y.

(i) Write her utility function. **[Eco. (H) III Sem. 2018]**

(ii) If price of good X is Rs. 5 and price of good Y is Rs. 10, he has to spend all his money income of Rs. 200 on good X and Y only. Find the optimal consumption of X and Y. Illustrate diagrammatically.

[**Ans.** (i) U = min (2X, Y), (ii) X* = 8, Y* = 16]

Income Tax versus Quantity Tax

Q56. An agent's utility function is written as $U = 12X^2Y^2$, and his budget constraint is $X + 3Y = 120$. What are the optimal amounts of X and Y ? The income tax is definitely superior to the quantity tax in the sense that you can raise the same amount of revenue from a consumer and still leave him or her better off under the income tax than under the quantity tax. Verify this result for a quantity tax t = 2 on X.

Ans. Without tax X = 60, Y = 20 & U = 1,72,80,000; With quantity tax X = 20, Y = 20 & U = 19,20,000; with income tax X = 40, Y = 40/3 & U = 34,13,333.33

Q57. The utility function of Ayush is $U(x, y) = x^{1/2} y^{1/2}$, $p_x = 2$ and $p_y = 1$ and income of Ayush is 120.

(i) Find his optimum consumption bundle and level of utility.
(ii) If govt. imposes a quantity tax of Rs. 1 per unit on good X, then find his new optimum consumption bundle and level of utility.
(iii) Show that by imposing income tax rather than quantity tax the govt. can collect same revenue but with greater utility for Ayush.

Ans. **(i)** $x = 30$, $y = 60$ and U = $\sqrt{1800}$, **(ii)** $x = 20$, $y = 60$, U = $\sqrt{1200}$ & R* = 20
(iii) $x = 25$, $y = 50$, U = $\sqrt{1250}$ & R* = 20

Q58. Assuming that a consumer consumes both goods (x and y), give an example of the kind of preferences that will make him just as well off facing a quantity tax on commodity x, as an income tax, where both taxes help the government raise the same amount of tax revenue? Explain with the help of a diagram. **[Eco. (H) III Sem. 2014]**

MA Entrance

Q1. A consumer with utility function $u(x, y) = 2x + y$, income of Rs. 100, and facing price of Re. 1 each for both goods, will maximize utility

(a) At all points (x, y) that lie on the budget line.
(b) At $x = 50$, $y = 50$.
(c) By buying only y. (d) By buying only x.

[**Ans. :** (c)] **[DSE MA Ent. Eco. 1999]**

Q2. Red apples and green apples are perfect substitutes for Sheila. Price of red apple is Rs. 20 per kg and price of green apple is Rs. 18 per kg. In equilibrium, Sheila consumes **[DSE MA Ent. Eco. 1999]**

(a) Only green apples (b) Only red apples
(c) Both the varieties but consumes more green apples than red.
(d) None of the above is necessarily true.

[**Ans. :** (a)]

Q3. Suppose a consumer's preferences display increasing marginal rates of substitution between goods X and Y. If the prices of the two commodities and consumer's income are given, then **[DSE MA Ent. Eco. 2000]**

(a) The consumer will never maximize his utility.
(b) The consumer will choose to consume nothing at all.
(c) There will be a corner solution.
(d) None of the above.

[**Ans. :** (c)]

Q4. Commodities x_1 and x_2 are perfect complements, and the utility function is given by U = min $\{x_1, 3x_2\}$. (That is, U = x_1 if $x_1 \leq 3x_2$, U = $3x_2$ if $x_1 \geq 3x_2$). Price of x_1 is 2, price of x_2 is 1, and income is 140. At equilibrium, the consumer consumes
(a) Zero units of x_1 (b) Zero units of x_2
(c) 90 units of x_1 and 30 units of x_2 (d) 60 units of x_1 and 20 units of x_2
[**Ans. :** (d)] **[DSE MA Ent. Eco. 2001]**

Q5. If the indifference curve is strictly concave (bowed out from the origin) then in equilibrium **[DSE MA Ent. Eco. 2001]**
(a) Marginal rate of substitution will necessarily be equal to the price ratio.
(b) Only one of the commodities will be consumed.
(c) The consumer will minimize her utility.
(d) None of the above will be true.
[**Ans. :** (a)]

Q6. Sameer chooses to spend his entire income on food and buys no clothing. His indifference curves must therefore : **[DSE MA Ent. Eco. 2003]**
(a) Be vertical (b) Be horizontal
(c) Have a MRS greater than the price ratio where his budget line intersects the food axis.
(d) Have a MRS less than the price ratio where his budget line intersects the food axis.
[**Ans. :** (c)]

Q7. Antara has a utility function $U(x_1, x_2) = x_1.x_2$. She has 100 rupees to spend on these two commodities. If the price of x_1 is 5 and the price of x_2 is 10, then she will
(a) Definitely spend all her income of x_2.
(b) Definitely spend all her income of x_1.
(c) Purchase 10 units of x_1 and 5 units of x_2.
(d) Consume equal amounts of x_1 and x_2.
[**Ans. :** (d)] **[DSE MA Ent. Eco. 2003]**

Q8. Sunil's utility function is $U(x, y) = \min(x, y^2)$. If the price of x is 15, the price of y is 10 and Sunil choose to consume 7 units of y, Sunil's income must be :
(a) 175 (b) 905 (c) 805
(d) Cannot say on the basis of this information.
[**Ans. :** (c)] **[DSE MA Ent. Eco. 2003]**

Q9. Consider Ms Bijlee's utility function is min {E, W}, where E is her electricity consumption and W is her consumption of widgets. Suppose Ms Bijlee's income is 10 and the prices of widgets and electricity are 1. In order to curb Ms. Bijlee's electricity consumption, the electricity company decides to impose a surcharge of Re. 1 on every unit of electricity consumed in excess of 4 units. What is resulting reduction in Ms. Bijlee's electricity consumption? **[DSE MA Ent. Eco. 2005]**
(a) ¼ (b) 1/5 (c) 1/3 (d) 1/6
[**Ans. :** (c)]

Q10. Suppose Asha's preferences between two commodities x_1 and x_2 can be represented by $u(x_1, x_2) = \min\{x_1 - 5, x_2 + 3\}$. Given an income of Rs. 73, and facing prices of Rs. 3 for x_1 and Rs. 4 for x_2, Asha's optimal consumption bundle of (x_1, x_2) will be **[DSE MA Ent. Eco. 2006]**

(a) (12.5, 4.5) (b) (10.42, 10.42) (c) (15, 7) (d) (3, 16)

[**Ans. :** (c)]

Q11. Consider consumer having utility function $U = 2x + y$ where x and y are two commodities. Suppose that the price of x and y are both Rs. 5 each and the consumer's income is Rs. 100. In equilibrium, the consumer will consume the bundle **[DSE MA Ent. Eco. 2007]**

(a) (10, 10) (b) (0, 20) (c) (20, 0) (d) None of the above

[**Ans. :** (c)]

Q12. Suppose a consumer has a utility function $U = \min\{x + y, 2y\}$. He maximizes his utility subject to his budget constraint and consumes (x^*, y^*) = (3, 3). Which one of the following statements must be true? **[DSE MA Ent. Eco. 2008]**

(a) Price of good x is necessarily equal to price of good y.

(b) Price of good x is double the price of good y.

(c) Price of good x is less than or equal to price of good y.

(d) None of the above.

[**Ans. :** (c)]

Q13. A consumer spends an income of Rs. 100 on two goods, dosas and pizzas. Let x denote the number of dosas and y the number of pizzas consumed (fractions allowed). The consumer's utility function is $U = e^{x^2 + y^2}$. If the price of a dosa is Rs. 5, and the price of a pizza is Rs. 10, then the number of pizzas this consumer will buy is **[DSE MA Ent. Eco. 2009]**

(a) 0 (b) 10 (c) 5 (d) 8

[**Ans. :** (a)]

Answer the Questions 14 and 15 on the basis of the following information :

Suppose that a typical graduate student at the Delhi School of Economics lives in a two good world, books (x) and movies (y), with utility function $u(x, y) = x^{1/5}y^{4/5}$. Prices of books and movies are 50 and 10 respectively. Suppose the University is considering the following schemes.

Scheme 1: 750 is paid as fellowship and additional 250 as book grant. Naturally, book grant can only be spent on books.

Scheme 2: 1000 as scholarship and gets one movie free on each book they purchase.

Believing that books and movies are perfectly divisible, compute the optimal consumption bundle under each scheme.

Q14. Optimal consumption bundle under scheme 1 is **[DSE MA Ent. Eco. 2009]**

(a) (4 books, 80 movies) (b) (5 books, 75 movies)

(c) (6.5 books, 57.5 movies) (d) (10 books, 50 movies)

[**Ans. :** (b)]

Q15. Optimal consumption bundle under scheme 2 is **[DSE MA Ent. Eco. 2009]**
(a) (4 books, 80 movies) (b) (4 books, 84 movies)
(c) (5 books, 75 movies) (d) (5 books, 80 movies)
[**Ans. :** (d)]

Q16. Suppose there are just two goods, say x_1 and x_2. Consider a consumer who chooses $x_2 = 0$ for all income levels $w > 0$ and all prices $p_1 > 0$ and $p_2 > 0$. These choices are consistent with the consumer **[DSE MA Ent. Eco. 2010]**
(a) having utility function $u(x_1, x_2) = x_1 + 2x_2$.
(b) having utility function $u(x_1, x_2) = 2x_1 + x_2$.
(c) lexicographically preferring x_2 to x_1.
(d) lexicographically preferring x_1 to x_2.
[**Ans. :** (d)]

Q17. A consumer has the utility function $u(x, y) = xy$. Suppose the consumer demands bundle (x^*, y^*). Now suppose the seller of good x offers a "buy one, get one free" scheme : for each unit of good x purchased, the consumer gets other unit of x for free. Given this scheme, suppose the consumer buys bundle (x_d, y_d) and gets an additional x_d for free. Which one of the following statements must be true?
(a) $x_d > x^*$ and $y_d > y^*$ (b) $x_d > x^*$ and $y_d = y^*$
(c) $x_d > x^*$ and $y_d < y^*$ (d) $x_d = x^*$ and $y_d = y^*$
[**Ans. :** (d)] **[DSE MA Ent. Eco. 2010]**

Q18. A consumer has utility function $u(x_1, x_2) = \min\{2x_1 + x_2, x_1 + 2x_2\}$. Her income is $y = 100$, the prices are $p_1 = 20$ and $p_2 = 30$. The amount of x_1 in the utility maximizing bundle is : **[DSE MA Ent. Eco. 2011]**
(a) 7 (b) 5 (c) 2 (d) 0
[**Ans. :** (c)]

Q19. Consider the same utility function and income as above, but suppose the prices are $p_1 = 10$ and $p_2 = 30$. Then the amount of x_1 in the utility maximizing bundle is :
(a) 10 (b) 5 (c) 2.5 (d) 0
[**Ans. :** (a)] **[DSE MA Ent. Eco. 2011]**

Q20. Utility of a consumer is given by $u(x_1, x_2) = \min\{x_1, x_2\}$. His income is M, and price of good 2 is 1. There are two available price schemes for good 1: (i) per unit price 2 and (ii) a reduced per unit price 2 - θ along with a fixed fee T. A consumer would be indifferent between the above schemes if **[DSE MA Ent. Eco. 2013]**
(a) θ = 2T/M (b) θ = 3T/M (c) θ = T/M (d) θ = (T+1)/M
[**Ans. :** (b)]