Chapter 2

Theory of Probability

Learning Objectives :

After learning this chapter you will understand :

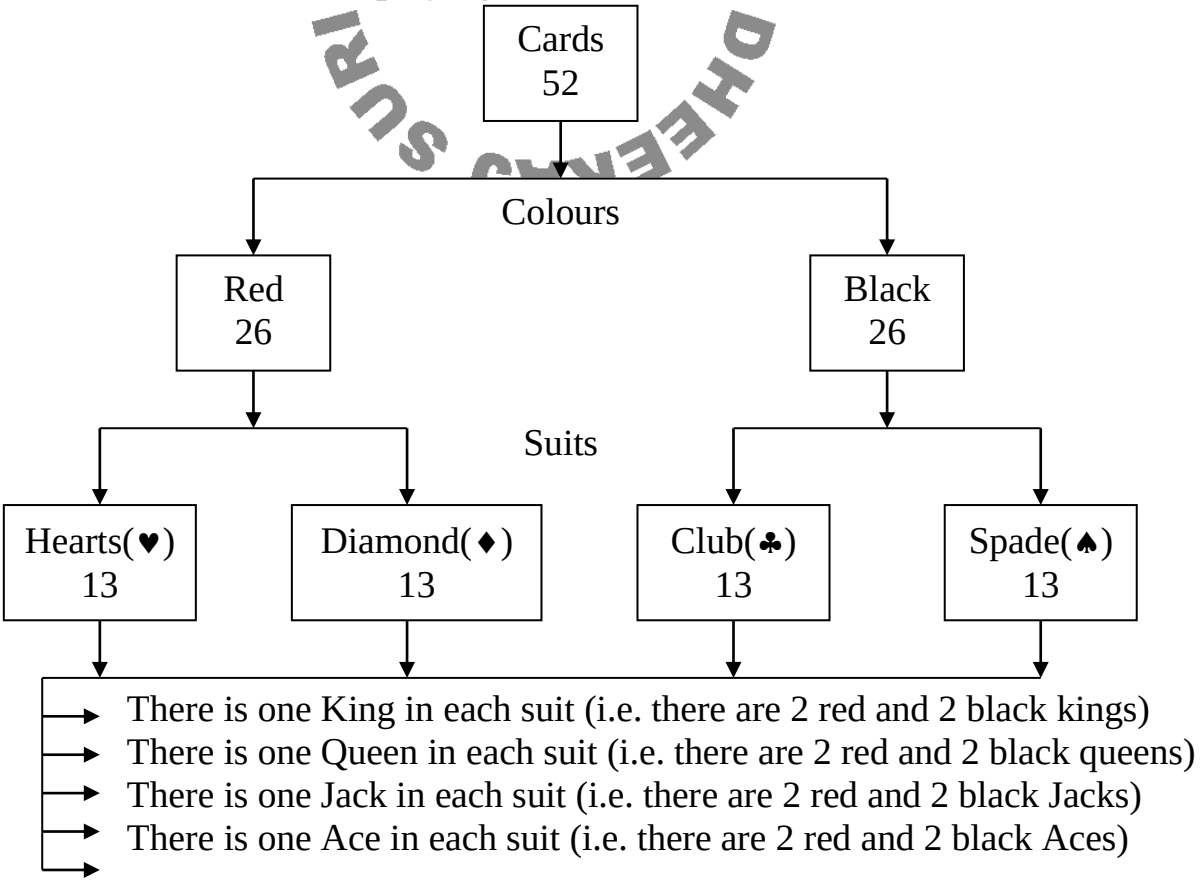
- **Random Experiment.**
- **Sample Space.**
- **Algebra of Events.**
- **Probability of an Event.**
- **Theorems of Probability**
  - ✓ **Addition Theorem**
  - ✓ **Multiplication Theorem**
- **Conditional Probability.**
- **Baye's Theorem.**
- **Mathematical Expectation.**

Basic Concepts

1. **Experiment :** An operation which results in some well defined outcomes is called an experiment.
2. **Random Experiment :** If an experiment has more than one possible outcomes, and when repeated under identical conditions, do not produce the same result every time but the outcome in a trial is one of the several possible outcomes, then such an experiment is known as random experiment.

> *Note :* An experiment whose outcome can be foretold before hand is not a random experiment.

3. **Trial :** When a random experiment is repeated under identical conditions and it does not give the same result each time but may result in one of the several possible outcomes, then such experiment is called a trial. e.g. tossing a coin or rolling a die. The number of times the experiment is repeated is called number of trials.
4. **Sample Space :** The set of all possible outcomes of a random experiment is called its sample space. It is usually denoted by S. e.g. the sample space of tossing a coin is S = { H, T}
5. **Discrete Sample Space :** A sample space S is called a discrete sample space if S is a finite set.
6. **Some Information about Die :** A die is a cube with six faces such that each face bears one of the numbers from 1, 2, 3, 4, 5, 6.
7. **Some Information about playing cards :**



There are 9 number cards in each suit.

Algebra of Events

8. **Event :** Every non-empty subset A of the sample space S, is called an event. e.g. appearance of Heads on tossing a coin is an event.
9. **Occurrence of an Event :** An event is said to be occurred when the outcome of a trial fully satisfies the condition mentioned in the event.
10. **Simple Event :** A singleton subset of sample space S is called a simple event or elementary event.
11. **Compound Event :** A subset of the sample space S which contains more than one element is called a compound event.
12. **Impossible Event :** The null set ϕ is a subset of S, ϕ is thus an event known as impossible event.
13. **Sure Event :** The event S is known as sure event.
14. **Mutually Exclusive Events :** Those events which cannot occur together are called mutually exclusive events. e.g., A and B are two mutually exclusive events if $A \cap B = \phi$.

> *Note :* Simple events of a sample space are always mutually exclusive.

15. **Exhaustive Events :** For a random experiment A, set of events is said to be exhaustive if one of them must essentially happen every time the experiment is performed. e.g., if events $E_1, E_2, E_3, \ldots E_n$ are subsets of a sample space S then events $E_1, E_2, E_3, \ldots E_n$ are mutually exhaustive if "$E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$".
16. **Mutually Exclusive and Exhaustive Events :** Let S be the sample space associated with a random experiment, and let $E_1, E_2, E_3, \ldots E_n$ be the subsets of S such that $E_i \cap E_j = \phi$ for $i \neq j$, and $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$, then the collection of events $E_1, E_2, E_3, \ldots E_n$ is said to form a mutually exclusive and exhaustive system of events.

> *Note :* If events A and B are mutually exclusive and exhaustive then
> $A \cap B = \phi$ and $A \cup B = S$

17. **Equally likely Events :** Events are said to be equally likely if the happening of one of them is not expected in preference to others.
18. **Complement of an Event :** For a random experiment let S be the sample space then complement of event A means set of all those elements which are not in A and is denoted by A' or A^c or A or not A.

> *Note :* Events A and A' are mutually exclusive and exhaustive events.

> *Note :* Event A' occurs if and only if event A does not occur.

19. **Algebra of Events :** Assuming that the sample space is universal set, we use the following results in the theory of probability
    (i) $(A \cup B)$ or (A or B) is the event representing " either A or B". i.e., it denotes the occurrence of at least one of the events from A and B.

(ii) $(A \cap B)$ or (A and B) is the event representing " both A and b". i.e., it denotes the occurrence of both of the events A and B.

(iii) A' is the event representing " not A"

(iv) (A – B) is the event representing " A but not B"

S. No.	Verbal Description of events	Set Notation
1.	A or B (at least one of A or B)	$A \cup B$
2.	A and B (common to both A and B)	$A \cap B$
3.	Not A	A'
4.	A but not B	$A \cap B'$
5.	Neither A nor B	$A' \cap B'$
6.	At most one of A, B	$A' \cup B'$
7.	Exactly one from A and B	$(A \cap B') \cup (A' \cap B)$
8.	At least one of A, B or C	$A \cup B \cup C$
9.	All three of A, B and C	$A \cap B \cap C$

20. **Probability of an Event :** If a trial results in *n* exhaustive, mutually exclusive and equally likely cases and *m* of them are favourable to the happening of an event A then the probability of happening of A is given by :

$$P(A) = \frac{\text{No. of favourable cases to A (i.e. } m\text{)}}{\text{No. of exhaustive cases (i.e. } n\text{)}} = \frac{n(A)}{n(S)}$$

Note : Probability of occurrence of an event is a number lying between 0 and 1. i.e. $0 \le P(A) \le 1$.

21. **Odds in favour of an event :** If odds in favour of occurrence of an event A are a : b then $P(A) = \frac{a}{a+b}$ and $P(A') = \frac{b}{a+b}$.

22. **Odds against an event :** If odds against the occurrence of an event A are a : b then $P(A) = \frac{b}{a+b}$ and $P(A') = \frac{a}{a+b}$.

Note : Odds in favour of an event = $\frac{\textit{Number of favourable outcomes}}{\textit{Number of unfavourable outcomes}}$ i.e. $\frac{n(A)}{n(s)-n(A)}$.

Note : Odds against an event = $\frac{\textit{Number of unfavourable outcomes}}{\textit{Number of favourable outcomes}}$ i.e. $\frac{n(s)-n(A)}{n(A)}$.

Note : Probability of an *impossible event* is equal to 0, i.e., $P(\phi) = 0$.

Note : Probability of a *sure event* is equal to 1, i.e., P(S) = 1.

Note : The sum of probabilities of mutually exclusive and exhaustive events is equal to 1. i.e. if E_1, E_2, E_3,….E_n are mutually exclusive and exhaustive events then $P(E_1) + P(E_2) + P(E_3) + \ldots\ldots + P(E_n) = 1$.

23. **Steps required to find the probability of an event :**

(i) Construct the sample space of the given random experiment and denote it by S.

(ii) Define the events whom probability is to be found and denote them by A, B etc.

(iii) Find the number of elements favourable to the event from the sample space, i.e. find n(A).

(iv) Find the probability of the event defined above using the formula $P(A) = \frac{n(A)}{n(s)}$.

Note : Generally the sample space should be constructed only for the questions involving coins and dice.

Example 1 : Find the probability of getting two heads if three coins are tossed simultaneously.

Solution : Let S be the sample space of the experiment

S = {HHH,HHT,HTH,THH,HTT,THT,TTH,TTT}

Let A : Two heads appear

A = {HHT,HTH,THH}

$\therefore$ n(A) = 3 and n(S) = 8

Now, $P(A) = \frac{n(A)}{n(s)} = \frac{3}{8}$

Exercise 1

Q1. If S is the sample space of an experiment, then what is P(S).

Q2. A die is tossed. What is the probability of getting an even number.

Q3. A card is drawn from a well shuffled pack of 52 cards. Find the probability that the card is :

(i) a king, (ii) a spade, (iii) a red card, (iv) a queen of hearts, (v) not a face card

Q4. A bag contains 10 pencils of which 4 are blue, 3 are red 2 are green and 1 is pink. A pencil is drawn from the bag find the probability that it is

(i) blue, (ii) red, (iii) green, (iv) pink (v) not red

Q5. 50 cards are numbered 1 to 50. If one card is drawn at random, what is the probability that it is a multiple of 6.

Q6. A box contains 6 red marbles numbered from 1 to 6 and 4 white marbles numbered from 12 to 15. Find the probability that a marble drawn at random is

(i) white and odd numbered, (ii) red or even numbered

Q7. The odds in favour of an event are 3 : 4 find the chance that it will occur.

Q8. The odds in favour of occurrence of an event are 5 : 13. Find the probability that it will occur.

Q9. The odds against an event are 4 to 5. Find the probability of its occurrence.

Q10. A card is drawn from a well shuffled deck of cards. What are the odds in favour of getting a spade.

Q11. A card is drawn from a well shuffled deck of cards. What are the odds in against getting a king.

Q12. Two dice are thrown. Find the odds in favour of getting a sum of 6.

Q13. In a throw of two coins, find the probability of getting both heads or both tails.

Q14. Find the probability of getting two heads if three coins are tossed simultaneously.

Q15. In tossing a coin thrice, find the chance of throwing head and tail alternately in three successive trials.

Q16. Three perfect coins are tossed together. What is the probability of getting at least one head?

Q17. A coin is tossed successively 3 times. Find the probability of getting exactly one head or two heads.

Q18. If four coins are tossed once simultaneously, find the probability of getting two heads and two tails.

Q19. Four coins are thrown simultaneously. What is the probability of getting at least one head.

Q20. In a single throw of two dice, find the probability of getting a doublet of even number.

Q21. In a single throw of two dice, what is the probability of obtaining ' a total of 9'?

Q22. A pair of dice is thrown, find the probability that the sum of the numbers obtained is more than ten.

Q23. In the play of two dice, the thrower loses if his first throw is 2, 4 or 12. He wins if his first throw is 5 or 11. Find the ratio between his probability of losing and probability of winning in the first throw.

Q24. Two dice are thrown together. What is the probability that the sum of the number on the two faces is greater than 8.

Q25. Two unbiased dice are thrown. Find the probability of getting a sum more than 7.

Q26. Two dice are thrown simultaneously. Find the probability of getting prime numbers on each die.

Q27. Two dice are thrown. Find the probability of getting an odd number on one die and a multiple of 3 on the other.

Q28. In a single throw of 2 dice, find the probability of obtaining a total of 9 or 11.

Q29. Find the probability of getting the sum as a prime number when two dice are thrown together.

Q30. Two dice are thrown simultaneously. Find the probability of getting a multiple of 3 as the sum.

Q31. Two dice are thrown simultaneously. Find the probability of getting six as the product.

Q32. Find the probability of getting the product as a perfect square (square of a natural number) when two dice are thrown together.

Q33. Find the probability that a leap year contains 53 Sundays. **[Eco (H) 2008]**

Q34. A bag contains 3 red balls bearing numbers 1, 2, or 3(one number on one ball); and 2 black balls bearing the numbers 4 or 6. A ball is drawn, its number is noted and the ball is replaced in the bag. Then another ball is drawn and its number is noted. Find the probability of drawing
(i) 2 on first draw and 6 on second draw
(ii) a number less than or equal to 2 and 4 on second draw
(iii) a total of 5

Q35. In a city, 30% of households read a Newspaper A. 50% read Newspaper B. 12% read Newspaper A and B. 25% read only India Today (Say C). 5% read all the three (A, B, C). Out of A readers 20% read C. Out of B readers 32 % read C
Using a Venn diagram of three intersecting sets, find the probability of a household, selected at random reading:
(i) At least one of all the three;
(ii) None of these. **[Eco (H) 2001]**

Q36. A seller has a dozen of water meters, two of which are faulty. A customer is interested in buying all the meters. The seller can package all the meters in one large box or six each in two identical smaller boxes. If the customer is likely to inspect two meters from a large box and one meter each from the smaller boxes, which of the following strategies should the seller adopt in his attempt to sell the faulty meters without being detected?
(i) pack all 12 meters in one large box;
(ii) put one faulty meter in each smaller box;
(iii) put both the faulty meters in one of the smaller boxes and no faulty ones in the other. **[Eco. (H) I Sem. 2011]**

Q37. Let A, B, C be three distinct events defined on sample space S. Using Venn diagram arrange the probability of the following events from the smallest to the largest **[Eco. (H) I Sem. 2014]**

$A \cup B$, $A \cap B$, A, S, $(A \cap B) \cup (A \cap C)$

MA Entrance

Q1. If two fair dice are tossed, the probability that the sum of the points on the dice equal 7 is **[DSE MA Ent. Eco. 2000]**
(a) ½ (b) 3/6 (c) 2/6 (d) 1/6
[**Ans. :** (d)]

Q2. If a fair coin is tossed twice, the probability of getting 'head' at least once is
(a) ½ + ½ (b) ½ × ½ (c) ¾ (d) 2/4
[**Ans. :** (c)] **[DSE MA Ent. Eco. 2000]**

Q3. A die is loaded in such a way that the probability of each face turning up is proportional to the number of dots on that face. (For example, a six is three times as probable as two). What is the probability of getting an even number in one throw? **[DSE MA Ent. Eco. 2001]**

(a) 1/3 (b) 12/21 (c) 9/21 (d) 13/20
[**Ans. :** (b)]

Q4. If a point is taken at random from within a circle with radius *r*, what is the probability that its distance from the centre will exceed *r*/2.
(a) ½ (b) ¾ (c) π/2 (d) None of the above
[**Ans. :** (b)] **[DSE MA Ent. Eco. 2002]**

Q5. Three distinct points X, Y and Z are taken at random on a line segment. What is the probability that Z lies between X and Y? **[DSE MA Ent. Eco. 2002]**
(a) 1/3 (b) ½ (c) 1/9 (d) Cannot be evaluated
[**Ans. :** (a)]

Q6. Suppose we flip a fair coin three times. The probability of getting two heads is
(a) 2/8 (b) 1/3 (c) 2/3 (d) 3/8
[**Ans. :** (d)] **[DSE MA Ent. Eco. 2006]**

Q7. Consider an experiment in which five fibers having different lengths are subjecting to a testing process to learn which fiber will break first. The lengths of the five fibers 1 inch, 2 inches, 3 inches, 4 inches and 5 inches. Suppose the probability that any fiber will be the first to break is proportional to the length of that fiber. The probability that the length of the fiber that breaks first is not more than 3 inches is :
(a) 3/5 (b) 2/5 (c) 1/3 (d) 2/3
[**Ans. :** (b)] **[DSE MA Ent. Eco. 2007]**

Q8. A primary school has 6 classes, numbered from 1 to 6. Class 1 has twice as many students as each of the other classes. What is the probability that a randomly selected student is from an odd numbered class? **[DSE MA Ent. Eco. 2007]**
(a) 10/21 (b) 3/7 (c) 8/2l (d) 4/7
[**Ans. :** (d)]

Q9. If two balanced dice are rolled, the sum of dots obtained is even with probability :
(a) ½ (b) ¼ (c) 3/8 (d) 1/3
[**Ans. :** (a)] **[DSE MA Ent. Eco. 2013]**

Answers of Exercise 1

1.	1,	**2.**	1/2,	**3(i)**	1/13,	**(ii)**	¼,	**(iii)**	½,
(iv)	1/52,	**(v)**	9/13,	**4(i)**	2/5,	**(ii)**	3/10,	**(iii)**	2/10,
(iv)	1/10,	**(v)**	7/10,	**5.**	4/25,	**6(i)**	1/5,	**(ii)**	4/5,
7.	3/7,	**8.**	5/18,	**9.**	5/9,	**10.**	1 : 3,	**11.**	12:1,
12.	5 : 31,	**13.**	1/2,	**14.**	3/8,	**15.**	1/4,	**16.**	
17.	3/4,	**18.**	3/8,	**19.**	15/16,	**20.**	1/12,	**21.**	1/9,
22.	1/12,	**23.**		**24.**	5/18,	**25.**	5/12,	**26.**	¼,
27.	11/36,	**28.**	1/6,	**29.**	5/12,	**30.**	1/3,	**31.**	1/9,
32.	2/9,	**33.**	2/7,	**34(i)**	1/25,	**(ii)**	2/25,	**(iii)**	4/25
35.	(i) 93%,	**(ii)**	7%.	**36.**	II strategy,				

Basic Concepts

1. **Use of Combinations :** When two or more items are drawn "simultaneously or successively without replacement" then we use combinations.

Note : Combination is used for selection of items.

Note : Number of ways to select r items from n items = ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$.

Example 2 : From a pack of 52 playing cards, three cards are drawn at random. Find the probability of drawing a king, a queen and a jack.

Solution : Let S be the sample space

$\therefore$ n(S) = total number of ways of selecting 3 cards from 52 cards

$= {}^{52}C_3$

E : the event that out of 3 cards drawn one is king, one is queen and one is jack

$\therefore$ n(E) = ${}^{4}C_1 \times {}^{4}C_1 \times {}^{4}C_1$

Now, $P(E) = \frac{n(E)}{n(S)} = \frac{{}^{4}C_1 \times {}^{4}C_1 \times {}^{4}C_1}{{}^{52}C_3}$

$= \frac{4 \times 4 \times 4}{\frac{52 \times 51 \times 50 \times 49!}{3! \times 49!}} = \frac{4 \times 4 \times 4 \times 6}{52 \times 51 \times 50} = \frac{16}{5525}$

Exercise 2

Q1. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled deck of cards. Find the probability of exactly one ace.

Q2. A bag contains 5 green and 7 red balls. Two balls are drawn. What is the probability that one is green and the other is red.

Q3. A bag contains 8 green and 10 white balls. Two balls are drawn. What is the probability that one is green and other is white.

Q4. A bag contains 8 red, 3 white and 9 blue balls. Three balls are drawn at random from the bag. Determine the probability that none of the balls is white.

Q5. A bag contains tickets numbered 1 to 22. Two tickets are drawn. Find the probability that both the numbers are prime.

Q6. An urn contains 7 white and 3 red balls. Two balls are drawn together at random from this urn, find out the probability that :

(i) Neither of them is white ;

(ii) One ball is white and the other is red.

Q7. An urn contains 7 white, 5 black and 3 red balls. Two balls are drawn at random. Find the probability that

(i) both the balls are red (ii) one ball is red and other is black

(iii) one ball is white

Q8. An urn contains 6 white, 4 black and 5 red balls. Two balls are drawn at random. Find the probability that
(i) both the balls are red (ii) one ball is red and other is black
(iii) one ball is white

Q9. 3 cards are drawn at random from a pack of well shuffled 52 cards. Find the probability that
(i) all the three cards are of same suit
(ii) one is a king, the other is queen and the third is a jack.

Q10. From a bag containing 20 tickets, numbered 1 to 20, two tickets are drawn at random. Find the probability that
(i) both the tickets have prime numbers on them
(ii) on one there is prime number and on other there is multiple of 4.

Q11. Four cards are drawn at a time from a pack of 52 playing cards. Find the probability of getting all the four cards of the same suit.

Q12. Four cards are drawn at a time from a pack of 52 playing cards. Find the probability of getting all the four cards of the same number.

Q13. A bag contains 6 red, 4 green, 2 pink and 5 black balls. If 4 balls are drawn at random, find the probability of the balls beings of different colours.

Q14. Four cards are drawn at a time from a pack of 52 playing cards. Find the probability of getting 3 diamonds and 1 spade.

Q15. In a group of 10 outstanding students in a school, there are 6 boys and 4 girls, 3 students are to be selected out of these at random for a debate competition. Find the probability that one is boy and 2 are girls.

Q16. A committee of 5 Principals is to be selected from a group of 6 gents Principals and 8 lady principals. If the selection is made randomly, find the probability that there are 3 lady Principals and 2 gents principals.

Q17. Out of 9 outstanding students in a college there are 4 boys and 5 girls. A team 4 students is to be selected for a quiz program. Find the probability that 2 are girls and 2 are boys.

Q18. In a group there are 3 men and 2 women. 3 persons are selected at random from this group. Find the probability that 1 man and 2 women or 2 men and 1 woman are selected.

Q19. In a group there are 2 men and 3 women. 3 persons are selected at random from this group. Find the probability that 1 man and 2 women or 2 men and 1 woman are selected.

Q20. In a group there are 3 men and 3 women. 4 persons are selected at random from this group. Find the probability that 1 man and 3 women or 3 men and 1 woman are selected.

Q21. A bag contains 4 red, 3 black and 3 white balls. Two balls are drawn from the bag. What is the probability that none of the balls drawn is a white ball.

Q22. Four persons are chosen at random from a group consisting of 3 men, 2 women and 3 children. Find the probability that out of 4 chosen persons exactly 2 are children.

Q23. A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn at random from the box, what is the probability that
(i) all will be blue (ii) at least one will be green

Q24. 5 marbles are drawn from a bag which contains 7 blue marbles and 4 black marbles. What is the probability that
(i) All will be blue, (ii) 3 will be blue and 2 black?

Q25. Find the probability when a hand of 7 cards is dealt from a well shuffled deck of 52 cards, it contains
(i) 4 kings, (ii) exactly 3 kings, (iii) at least 3 kings

Q26. A class consists of 10 boys and 8 girls. Three students are selected at random. What is the probability that the selected group has at least one girl.

Q27. Suppose that a Central University has to form a committee of 5 members from a list of 20 candidates out of whom 12 are teachers and 8 are students. If the members of the committee are selected at random, what is the probability the majority of the committee members are students? **[Eco. (H) 2009]**

Q28. A committee of 6 people is to be chosen from a group consisting of 7 men and 8 women. If the committee must consist of at least 3 women and at least 2 men, how many different committees are possible? **[Eco. (H) I Sem. 2014]**

Q29. A bag contains 30 tickets, numbered from 1 to 30. Five tickets are drawn at random and arranged in ascending order. Find the probability that the third number is 20.

MA Entrance

Q1. A lot consists of 10 articles, 4 with minor defects and 2 with major defects. If 2 articles are picked at random, the probability that both have minor defects is
(a) ¼ (b) 1/8 (c) 1/20 (d) None of the above
[**Ans. :** (d)] **[DSE MA Ent. Eco. 2000]**

Q2. A box contains 10 fuses, 4 of which are defective. If 3 of the fuses are drawn without replacement, what is the probability that all three are defective?
(a) 1/30 (b) ¾ (c) 2/5 (d) 3/10
[**Ans. :** (a)] **[DSE MA Ent. Eco. 2002]**

Q3. There are three women on the platform of a train station. The train that they are waiting for has 5 coaches and each of them is equally likely to enter any coach. What is the probability that they will all enter the same coach?
(a) 12/25 (b) 3/5 (c) 3/125 (d) 9/25
[**Ans. :** (c)] **[DSE MA Ent. Eco. 2006]**

Q4. Suppose a neighborhood has 90 Hindus and 10 Muslims. What is the probability that two randomly selected persons from that neighborhood will have the same religion? **[DSE MA Ent. Eco. 2006]**
(a) 0.5 (b) 0.81 (c) 0.9 (d) 0.82
[**Ans. :** (d)]

Q5. Your teacher knows 6 jokes and in each class tells 2 jokes, each joke has an equal chance of being selected. What is the probability that, in a given lecture, at least 1 joke is told that was not told in the previous class? **[DSE MA Ent. Eco. 2006]**
(a) 28/30 (b) 14/30 (c) 16/30 (d) 12/30
[**Ans. :** (a)]

Answers of Exercise 2

1. $\frac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{32}{221}$, **2.** 35/66, **3.** 80/153, **4.** 34/57,

5. 4/33, **6.(i)** 1/15, **(ii)** 7/15,

7. **(i)** $\frac{{}^3C_2}{{}^{15}C_2} = \frac{1}{35}$, **(ii)** $\frac{{}^3C_1 \times {}^5C_1}{{}^{15}C_2} = \frac{1}{7}$, **(iii)** $\frac{{}^7C_1 \times {}^8C_1}{{}^{15}C_2} = \frac{8}{15}$,

8. **(i)** $\frac{{}^5C_2}{{}^{15}C_2} = \frac{2}{21}$, **(ii)** $\frac{{}^5C_1 \times {}^4C_1}{{}^{15}C_2} = \frac{4}{21}$, **(iii)** $\frac{{}^6C_1 \times {}^9C_1}{{}^{15}C_2} = \frac{18}{35}$,

9. **(i)** $4 \times \frac{{}^{13}C_3}{{}^{52}C_3} = \frac{22}{425}$, **(ii)** $\frac{{}^4C_1 \times {}^4C_1 \times {}^4C_1}{{}^{52}C_3} = \frac{16}{5525}$,

10. **(i)** 14/95, **(ii)** 4/19, **11.** 44/4165, **12.** 1 / 20825,

13. 12/119, **14.** 286/20825, **15.** 3/10, **16.** 60/143,

17. 10/21, **18.** 9/10, **19.** 9/10, **20.** 2/5,

21. 7/15, **22.** 800/2187, **23(i)** 34/11977, **(ii)** 4367 / 4484,

24(i) 1/22, **(ii)** 5/11, **25(i)** 1/7735, **(ii)** 9/1547,

(iii) 46/7735, **26.** 29/34, **27.** 0.3, **29.** $\frac{{}^{19}C_2 \times {}^{10}C_2}{{}^{30}C_5} = \frac{285}{5278}$

Basic Concepts

1. **Use of Permutations :** When two or more items are to be arranged then we use permutations.

Note : When order is important in given items then we use permutations.

Note : Number of ways to arrange *r* items from *n* items = $^nP_r = \frac{n!}{(n-r)!}$.

Note : The total number of permutation of n items of which all are not distinct and where p_1 are alike items of one kind, p_2 are alike items of second kind,…..p_r are alike items of r^{th} kind = $\frac{n!}{(p_1!)\times(p_2!)\times\ldots(p_r!)}$.

Example 3 : Four digit numbers are formed by using the digits 1, 2, 3, 4 and 5 without repeating any digit. Find the probability that the number is divisible by 4.

Solution : Any four digit number is divisible by 4 if its last two digits are from the set {12, 24, 32, 52}

$\therefore$ required probability = $\frac{{}^4C_1 \times {}^3P_2}{{}^5P_4} = \frac{\frac{4!}{3!} \times \frac{3!}{1!}}{\frac{5!}{1!}} = \frac{4\times3\times2}{5\times4\times3\times2} = \frac{1}{5}$

Exercise 3

Q1. Four digit numbers are formed by using the digits 1, 2, 3, 4 and 5 without repeating any digit. Find the probability that a number, choosen at random is an odd number.

Q2. Four digit numbers are formed by using the digits 1, 2, 3, 4 and 5 without repeating any digit. Find the probability that the number is divisible by 4.

Q3. The letters of the word 'SOCIETY' are placed at random in a row. What is the probability that three vowels come together.

Q4. The letters of the word 'FORTUNATES' are arranged at random in a row. What is the chance that the two 'T' come together.

Q5. The letters of the word 'UNIVERSITY' are arranged at random in a row. What is the probability that the two I's always come together.

Q6. The letters of the word 'MATHEMATICS' are placed at random in a row. What is the probability that two A's come together ?

Q7. If 10 men, among whom are A and B, stand in a row, what is the probability that there will be exactly 3 men between A and B? **[Eco. (H) 2002]**

MA Entrance

Q1. Each of the four economists is asked to prescribe one out of four economic policies. Each economist is equally likely to prescribe any of the four different policies. What is the probability that each of the economists prescribes a different policy? **[DSE MA Ent. Eco. 2004]**

(a) 3/8 (b) 1/256 (c) 3/16 (d) 3/32

[**Ans. :** (d)]

[**Hint :** Total chances $4^4 = 256$, favourable chances $= {}^4P_4 = 24$]

Q2. Suppose that each person out of a group of 4 persons is randomly assigned to one of 6 classes. What is the probability that no class has more than one person from this group? **[DSE MA Ent. Eco. 2007]**

(a) 6!/4! (b) 5/18 (c) 4/6 (d) 6/4!

[**Ans. :** (b)]

Q3. The nine digits 1, 2,…, 9 are arranged in random order to form a nine digit number, which uses each digit exactly once. Find the probability that 1, 2 and 3 appear as neighbours in the increasing order. **[DSE MA Ent. Eco. 2009]**

(a) 1/12 (b) 1/72 (c) 1/84 (d) $(2/3)^9$

[**Ans. :** (b)]

Q4. Two women and four men are to be seated randomly around a circular table. Find the probability that the women are not seated next to each other.

(a) ½ (b) 1/3 (c) 2/5 (d) 3/5

[**Ans. :** (d)] **[DSE MA Ent. Eco. 2012]**

Answers of Exercise 3

1. 3/5, **2.** 1/5, **3.** 1/7, **4.** 1/5, **5.** 1/5, **6.** 2/11

Basic Concepts

Theorems of Probability

1. **Addition Theorem of Probability :**
   $P(A \cup B)$ or $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$
2. If A and B are mutually exclusive events then $P(A \cap B) = 0$. Thus,
   $P(A \cup B) = P(A) + P(B)$

> *Note :* $P(A \cup B)$ or $P(A \text{ or } B)$ represents the probability of occurrence of at least one of the events A and B.

> *Note :* $P(A \cap B)$ or $P(A \text{ and } B)$ represents the probability of occurrence of both of the events A and B.

3. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
4. If A, B and C are mutually exclusive events then $P(A \cap B) = P(B \cap C) = P(C \cap A) = P(A \cap B \cap C) = 0$. Thus, $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
5. $P(A) + P(\bar{A}) = 1$, where $P(\bar{A})$ is the probability of the non occurrence of event A.
6. $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

> **Trick :** To find **P(neither A nor B)** use *P(neither A nor B) = 1 – P(A or B)*

7. $P(A \cup \bar{B}) = P(\bar{B}) + P(A \cap B)$
8. $P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$
9. $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

Example 4 : Two cards are drawn at random from a pack of cards. Find the probability that both the cards are of red colour or they are queen.

Solution : Let S be the sample space of the experiment of drawing 2 cards from a pack of 52 cards.
and A : the event that two cards are red
B : the event that two cards are queens
$\therefore$ n(S) = ${}^{52}C_2$, n(A) = ${}^{26}C_2$ and n(B) = ${}^{4}C_2$
Now, required probability = P(A or B)
= P(A) + P(B) – P(A ∩ B)

$$= \frac{{}^{26}C_2}{{}^{52}C_2} + \frac{{}^{4}C_2}{{}^{52}C_2} - \frac{{}^{2}C_2}{{}^{52}C_2} \qquad = \frac{26\times25}{52\times51} + \frac{4\times3}{52\times51} - \frac{2\times1}{52\times51} = \frac{285}{2652}$$

Exercise 4

Q1. Find P(A or B) if P(A) = 2/3, P(B) = 7/15 and P(A and B) = 1/5.
Q2. If E and F are two events associated with a random experiment for which P(E) = 0.60, P(E or F) = 0.85, P(E and F) = 0.42, find P(F).
Q3. If P(A) = ¼, P(B) = ½, and P(A and B) = 1/8. Find P(not A and not B).
Q4. Events A and B are such that P(A) = ½, P(B) = 7/12 and P(not A or not B) = ¼. State whether A and B are mutually exclusive.

Q5. A and B are two mutually exclusive events. If $P(\bar{A}) = 0.5$, $P(B) = 0.4$, find P(A or B).

Q6. A, B, C are three mutually exclusive and exhaustive events associated with a random experiment. Find P(A), it being given that $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$.

Q7. Comment on the following :
Given : $P(A) = ½$, $P(B) = 3/8$ and $P(A \cap B) = ¼$; $P(A \cup B) = 5/8$.

Q8. If A and B are mutually exclusive events associated with a random experiment such that P(A) = 0.4 and P(B) = 0.5, then find
(i) $P(A \cup B)$, (ii) $P(\bar{A} \cap \bar{B})$, (iii) $P(\bar{A} \cap B)$, (iv) $P(A \cap \bar{B})$

Q9. Given P(A) = 0.35, P(B) = 0.73 and $P(A \cap B) = 0.14$. Find :
(i) $P(\bar{A} \cap B)$ (ii) $P(A \cap \bar{B})$ (iii) $P(A \cup B)$ (iv) $P(\bar{A} \cup \bar{B})$
[Eco. (H) II Sem. 2014]

Q10. A coin is tossed 4 times and the resulting sequence of heads and tails is recorded. The events A, B & C are defined as follows : **[Eco. (H) I Sem. 2012]**
A : Exactly 2 heads
B : Head Tails alternate
C : First 2 tosses are heads
(i) What elements constitute the sample space of the events A, B and C?
(ii) Which events, if any, are mutually exclusive?
(iii) Which events, if any, are subsets of other sets?

Q11. Is this statement true. Explain your answer.
The probability that Team A will win a cricket match is 0.67, the probability that it will tie the game is 0.04 and the probability that it will win or tie the game is 0.85.
[Eco. (H) I Sem. 2012]

Q12. The probability that Rohan will get an award in mathematics is 0.18, the probability that he will get an award in statistics is 0.25, and the probability that he gets awards in both is 0.13. **[Eco. (H) II Sem. 2014]**
(i) What is the probability that he will get at least one of the two awards?
(ii) What is the probability that he will get only one of the two awards?

Q13. A die is thrown twice. Find the probability that the sum of the two numbers obtained is 5 or 7.

Q14. Two dice are tossed once. Find the probability of getting an even number on the first dice or a total of 8.

Q15. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is divisible by 3 or 4.

Q16. A die is thrown three times find the probability of getting a total of less than 5 or a triplet.

Q17. In a single throw of three dice, determine the probability of getting a total of at least 5.

Q18. An integer is choosen from first 200 positive integers. Find the probability that it is divisible by 6 or 8.

Q19. From a well shuffled pack of 52 cards, a card is drawn at random, find the probability that it is either a heart or a queen.

Q20. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither 9 nor 11.

Q21. Two unbiased dice are thrown. Find the probability that neither a doublet nor a total of 10 will appear.

Q22. Two unbiased dice are thrown. Find the probability that the sum of the numbers obtained on the two dice is neither a multiple of 2 nor a multiple of 3.

Q23. A Card is drawn at random from a well shuffled pack of 52 cards. Find the probability that it is neither an heart nor a king.

Q24. A drawer contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one of them is choosen at random, what is the probability that it is rusted or a bolt?

Q25. A basket contains 20 apples and 10 oranges, out of which 5 apples and 3 oranges are rotten. If a person picks up two fruits from the basket at random, find the probability that either both are apples or both are good.

Q26. A bag contains 5 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that both balls are red or both are black?

Q27. There are two bags, one of which contains 3 red and 5 white balls and the other contains 5 red and 6 white balls. Two balls are drawn from one of the bags at random. Find the probability of drawing a red and a white ball.

Q28. An integer is chosen at random from the numbers ranging 1 to 50. What is the probability that the integer chosen is a multiple of 2 or 3 or 5?

Q29. The probabilities that a student will receive A, B, C or D grade are 0.40, 0.35, 0.15 and 0.10 respectively. Find the probability that a student will receive
(i) not an A grade, (ii) B or C grade, (iii) at most C grade

Q30. The probability that a student passes in mathematics is 2/3 and the probability that he passes in English is 4/9. If the probability of passing both courses is ¼, what is the probability that he will pass in at least one of these subjects ?

Q31. The probability that a patient visiting a dentist will have a tooth extracted is 0.06, the probability that he will have a cavity filled is 0.2 and the probability that he will have a tooth extracted as well as cavity filled is 0.03. What is the probability that a patient has either a tooth extracted or a cavity filled ?

Q32. The probability that a patient visiting a dentist will have a tooth cleaned is 0.44, the probability that he will have a cavity filled is 0.24. The probability that he will have a tooth cleaned or cavity filled is 0.60. What is the probability that the patient will have tooth cleaned and a cavity filled ?

Q33. A piece of electronic equipment has two essential parts, A and B. In the past, part A failed 30% of the times, part B failed 20% of the times and both the parts failed simultaneously 5% of the times. Assuming that both the parts must operate to enable the equipment to function, what is the probability that the equipment will function? **[Eco. (H) 1998, Eco. (H) ZHC 2009]**

Q34. From the employees of a company, 5 persons are elected to represent them in the managing committee of the company. Particulars of the five persons are as follows:

S.No.	Person	Age(in years)
1.	Male	30
2.	Male	33

3.	Female	46
4.	Female	28
5.	Male	41

A person is selected at random from this group to act as a spokesperson. What is the probability that that the spokesperson will be either male or over 35 years ?

Q35. The probability that a person will get an electric contract is 2/5 and the probability that he will not get plumbing contract is 4/7. If the probability of getting at least one contract is 2/3, what is the probability that he will get both ?

Q36. In a certain class it was found that 60% of the students passed in the terminal examination, 40% passed in the terminal and annual examination, 25 % passed in the annual but failed in the terminal examination. Find: The percentage of students who have passed (i) in the annual examination, (ii) in the terminal examination but failed in annual examination, (iii) failed in both the examinations. **[Eco (H) 2000]**

Q37. Prove that :

$$P(A_1 \cup A_2 \cup A_3 \cup .....A_n) \le P(A_1) + P(A_2) + P(A_3) + ... + P(A_n)$$

for any events $A_1, A_2, ...A_n$. **[Eco (H) 2002]**

Q38. An inspector of a pipeline company has the task of comparing two pumping stations. Each station is susceptible to two kinds of failure : pump failure and leakage. When either or both occur, the station must be shut down. The data at the hand indicate that the following probabilities prevail :

Station	P(pump failure)	P(leakage)	P(both)
1.	0.07	0.10	0
2.	0.09	0.12	0.06

Which station has the higher probability of being shut down? **[Eco. (H) 2011]**

Problems Involving Proofs

Q39. If $A_1, A_2, \ldots, A_n$ are independent events with respective probabilities of occurrence $p_1, p_2, ..., p_n$ then the probability of occurrence of at least one of them is given by:

$$P(A_1 \cup A_2 \cup .....\cup A_n) = 1 - (1 - p_1)(1 - p_2).....(1 - p_n)$$

MA Entrance

Q1. You toss a pair of fair dice. Let A be the event that the sum of numbers (on the top faces of the dice) is odd, B the event that at least one dice shows the number 1. Describe the events $A \cap B$, $A \cup B$, $A \cap B^c$. Find their probabilities. **[DSE MA Ent. Eco. 1999]**

Ans. $A \cap B$ = {(1, 2), (2, 1), (1, 4), (4, 1), (1, 6), (6, 1)}

$A \cup B$ = {(1, 2), (2, 1), (1, 4), (4, 1), (2, 3), (3, 2), (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3), (2, 7), (7, 2), (3, 6), (6, 3), (4, 5), (5, 4), (5, 6), (6, 5)}

$A \cap B^c$ = {(2, 3), (3, 2), (2, 5), (5, 2), (3, 4), (4, 3), (2, 7), (7, 2), (3, 6), (6, 3), (4, 5), (5, 4), (5, 6), (6, 5)}

Q2. If one integer is chosen at random from the sequence 1, ….. 50, the probability that the chosen number is divisible by 6 or 8 is **[DSE MA Ent. Eco. 2000]**

(a) 1/50 (b) 8/50 (c) 6/25 (d) 1/10

[**Ans. :** (c)]

Q3. Suppose four coins are tossed simultaneously. Suppose E is the event "the number of heads strictly exceeds the number of tails" and F is the event "the number of Tails strictly exceeds the number of Heads" what is the probability of the event $E \cup F$? **[DSE MA Ent. Eco. 2005]**

(a) 5/8 (b) ½ (c) ¾ (d) 3/8

[**Ans. :** (a)]

Q4. Suppose two dice are tossed simultaneously. What is the probability that the total number of spots on the upper faces of the two dice is not divisible by 2, 3 or 5?

(a) 1/3 (b) 2/9 (c) 4/9 (d) 7/16

[**Ans. :** (a)] **[DSE MA Ent. Eco. 2005]**

Q5. A survey of asset ownership in poor households in rural UP and Bihar found that 40% of the households own a radio, 15% own a television and 60% own a bicycle. It also found that 5% of the households own both a radio and a television, 26% own both a radio and a bicycle, 5% own both a television and a bicycle, and 1% own all three. If a randomly selected poor household in these areas is found to own exactly one of these three assets, what is the probability that it is a bicycle?

(a) 20/23 (b) 17/23 (c) 15/23 (d) 12/23

[**Ans. :** (c)] **[DSE MA Ent. Eco. 2008]**

Answers of Exercise 4

1.	14/15,	**2.**	0.67,	**3.**	3/8,	**4.**	No,	**5.**	0.9,
6.	4/13,	**8(i)**	0.9,	**(ii)**	0.1,	**(iii)**	0.5,	**(iv)**	0.4,
9.(i)	0.59,	**(ii)**	0.21,	**(iii)**	0.94,	**(iv)**	0.86,	**12.(i)**	0.30,
(ii)	0.17	**13.**	5/18,	**14.**	5/9,	**15.**	5/9,	**16.**	1/24,
17.	53/54,	**18.**	¼,	**19.**	4/13,	**20.**	5/6,	**21.**	7/9,
22.	1/3,	**23.**	9/13,	**24.**	5/8,	**25.**	316/435,		
26.	31/153,	**27.**	333/616,	**28.**	18/25,	**29(i)**	0.60,	**(ii)**	0.5,
(iii)	0.25	**30.**	31/36,	**31.**	0.23,	**32.**	0.08,	**33.**	0.55
34.	4/5,	**35.**	17/105,	**38.**	Station 1				

Basic Concepts

1. **Conditional Probability :** Let A and B be any two events associated with a random experiment where $B \neq \phi$, then the probability of occurrence of event A given that event B has already occurred is called the conditional probability and it is represented by P(A/B), i.e., P(A/B) implies the probability of occurrence of event A when B has already occurred. Likewise, P(B/A) implies the probability of occurrence of event B when A has already occurred.

Conditional Probability is found by using the following formula :

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \text{ and } P(B/A) = \frac{P(A \cap B)}{P(A)}$$

2. **Steps required to find conditional probability P(A/B) :**
   (i) construct the sample space of the given experiment (if possible).
   (ii) define the events A and B.
   (iii) identify the elements belonging to B from the sample space and hence find P(B).
   (iv) identify the elements belonging to A ∩ B from the sample space and hence find P(A ∩ B).
   (v) Use the formula $P(A/B) = \frac{P(A \cap B)}{P(B)}$ to find the conditional probability.

Note : Finding P(A/B) means computing P(A) from reduced sample space B, rather than from the original sample space S.
Note : If B = ϕ then B is an impossible event and the expression P(A/B) makes no sense.
Note : If A and B are mutually exclusive events then P(A/B) = 0.

Example 6 : If a die is thrown, what is the probability of appearance of a number greater than '1' if it is known that only odd numbers may appear.

Solution : Let S be the sample space of the experiment of throwing a die, then

S = {1, 2, 3, 4, 5, 6}

A : the event of appearance of a number greater than '1'

A = {2, 3, 4, 5, 6}

B : the event of appearance of odd number.

B = {1, 3, 5}

∴ A ∩ B = {3, 5}

So, n(S) = 6, n(A ∩ B) = 2, n(B) = 3

$\therefore P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{6}$ and $P(B) = \frac{n(B)}{n(S)} = \frac{3}{6}$

Now, required probability = P(A/B)

$= \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$

Exercise 5

Q1. If P(A) = $\frac{7}{13}$, P(B) = $\frac{9}{13}$, and P(A ∩ B) = $\frac{4}{13}$. Evaluate P(A | B).

Q2. If P(E) = 0.35, P(F) = 0.45 and P(E ∪ F) = 0.65, find P(F | E).

Q3. If P(E) = 0.40, P(F) = 0.35, P(E ∪ F) = 0.55 find P(F | E).

Q4. A and B are two events such that P(A) = ¼, P(B) = 1/3, and P(A ∪ B) = ½. Find P(B/A). **[Eco. (H) 2008]**

Q5. Let A and B be the two events such that P(A) = ½; P(B) = 1/3 and P(A ∩ B) = ¼. Obtain the probabilities : **[Eco. (H) 2008]**
(i) P(A/B), (ii) P(A ∪ B), (iii) $P(\overline{A} \cap \overline{B})$

Q6. If P(not A) = 0.7, P(B) = 0.7 and P(B/A) = 0.5 then find P(A/B) and P(A ∪ B).

Q7. Given: P(A) = 0.5 and P(A + B) = 0.7, find P(B) if : (i) A and B are independent events; (ii) A and B are mutually exclusive events; (iii) P(A/B) = 0.5. **[Eco. (H) 2005]**

Q8. Given that P(A) = 0.6 and P(A or B) = 0.8, find P(B) if :
(i) A and B are independent events,
(ii) A and B are mutually exclusive events,
(iii) P(A/B) = 0.5
Explain the rules of probability used in your answers. **[Eco. (H) I Sem. 2011]**

Q9. A dice is rolled. If the outcome is an even number what is the probability that it is a prime number.

Q10. A pair of dice is thrown. If the two numbers appearing on them are different. Find the probability that the sum of the numbers is 6.

Q11. A die is thrown twice and sum of the numbers is observed to be 7. What is the conditional probability that the number 2 has appeared at least once.

Q12. A die is thrown twice and sum of the numbers appearing to be 6. What is the conditional probability that the number 4 has appeared at least once.

Q13. A pair of dice is tossed. What is the probability that the sum obtained is 10 given that it exceeds 8? **[Eco. (H) I Sem. 2012]**

Q14. A pair of dice are tossed until a sum of 7 appears for the first time. What is the probability that more than four rolls will be required for that to happen? **[Eco. (H) I Sem. 2014]**

Q15. Two dice are thrown. If it is known that numbers on both the dice are different, find the probability that the sum of the two numbers is 4.

Q16. A die is thrown 3 times and the sum of the 3 numbers shown is 15. What is the probability that the number on the first throw is 4.

Q17. Two coins are tossed. What is the probability of coming up two heads if it is known that at least one head comes up.

Q18. A coin is tossed, then a die is thrown. Find the probability of obtaining a '6', given that heads come up.

Q19. Two integers are selected at random from integers 1 to 11. If the sum is even, find the probability that both the numbers are odd.

Q20. One card is drawn from a well shuffled pack of 52 cards. If E is the event "the card drawn is a king or queen" and F is the event "the card drawn is an ace or queen", then find the probability of the conditional event E/F.

Q21. One card is drawn from a well shuffled pack of 52 cards. If E is the event "the card drawn is a king or an ace" and F is the event "the card drawn is an ace or a jack", then find the conditional probability of the event E/F.

Q22. The probability that a student selected at random from a class will pass in mathematics is 4/5, and the probability that he/she passes in mathematics and

computer science is ½. What is the probability that he/she will pass in computer science if it is known that he has passed in mathematics.

Q23. In a certain school, 20% students failed in English, 15% failed in Mathematics and 10% students failed in both English and Mathematics. A student is selected at random. If he failed in English what is the probability that he also failed in Mathematics.

Q24. The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a trouser is 0.3, and the probability that he will buy a shirt given that he buys a trouser is 0.4. Find the probability that he will buy both a shirt and a trouser. Find also the probability that he will buy a trouser given that he buys a shirt.

Q25. A bag contains 3 red and 7 black balls. Two balls are selected at random without replacement. If the second ball is given to be red, what is the probability that the first ball is also red.

Q26. A couple has two children. Find the probability that both children are boys if it is known that at least one of the children is a boy.

Q27. An unbiased coin is tossed twice. **[Eco. (H) I Sem. 2011]**

(a) Define the sample space,

(b) Find the probability of two heads given at least one head.

Q28. Two digits are selected at random from the digits 1 through 9. Find :

(i) The probability that their sum is even. **[Eco. (H) 1996]**

(ii) If the sum is even, find the probability that both digits are odd.

Q29. Two fair dice are rolled. What is the probability that first die will show a number less than 3, given that the total for both dice will be less than 6?

[Eco. (H) II Sem. 2014]

Q30. Out of 5,000 people residing in a locality, 1200 are above 30 years of age and 3,000 are females. Out of 1200, who are above 30, 200 are females. Suppose after a person is chosen, you are told that the person so chosen is a female. What is the probability that she is above 30 years of age? Also explain the theory used for finding probability.

Q31. Each of the three identical jewellery boxes has 2 drawers. In each drawer of the first box there is gold watch. In each drawer of the second box there is a silver watch. In each drawer of the third box there is gold watch while in the other drawer there is a silver watch. If we select a box at random, open one of the drawers and find it to contain a silver watch, what is the probability that the other drawer has the gold watch? **[Eco. (H) 1994]**

Q32. Records of the performance of the particular type of electric generator show that the probability of failure in first 10 years of life of such a generator is 0.22. Given that the generator has failed, the conditional probability that the failure cannot be repaired is 0.45. Find the probability that such a generator will experience an irrepairable failure within its first 10 years of life.

Q33. Two boxes contain respectively 4 white and 2 black, and 1 white and 3 black balls. One ball is transferred from the first box into the second, and then 1 ball is drawn from the latter. It turns out to be black. What is the probability that the transferred ball was white? **[Eco. (H) II Sem. 2014]**

Q34. Based on a recent study it was found that 16% of all houses in a village own a refrigerator, 10% own a washing machine and 10% of those who own at least one in fact own both. If a randomly selected house is found to own a washing machine, what is the probability that it also owns a refrigerator? **[Eco. (H) I Sem. 2014]**

Q35. We have data from 100 economists in academic private sector and government concerning the opinions whether the economy would be stable expand or contract in the near future. However, a part of the information was lost. Based on the remaining data you are required to create the probability table :

Economists	Stable	Expanding(E)	Contracting	Total
Academics (A)	25		20	
Private Sector (R)		7		22
Govt. (G)	5	8		13
Total	40			

From the above table find :

(i) P(A), (ii) P(E), (iii) $P(A \cap E)$, (iv) $P(G \cap C)$,
(v) P(E/G), (vi) P(G/E). **[Eco. (H) 2006]**

Q36. A box of 100 gaskets contain 10 gaskets with type A defect, 5 gaskets with type B defect and 2 gaskets with both types of defect. Find the probability that :

(i) a gasket to be drawn has a type B defect under the condition that it has a type A defect; and
(ii) a gasket to be drawn has no type B defect under the condition that it has no type A defect.

Q37. During a survey of road safety, it was found that 60% of accidents occurs at night, 52% are alcohol related and 37% are alcohol related as well as occur at night.

(i) What is the probability that an accident was alcohol related given that it occurred at night?
(ii) What is the probability that an accident didn't occur at night, given that it was alcohol related?

Q38. Among the examinees in an examination 30%, 35% and 45% failed in Statistics, in Mathematics and in at least one of the subject respectively. An examinee is selected at random. Find the probability that :

(i) he failed in Mathematics only;
(ii) he passed in Statistics if it known that he failed in Mathematics.

Q39. The probability that a person stopping at a petrol pump will get his tyres checked is 0.12, the probability that he will get his oil checked is 0.29 and the probability that he will get both checked is 0.07.

(i) What is the probability that a person stopping at this pump will have neither his tyre checked nor oil checked.
(ii) Find the probability that a person who has oil checked, will also have his tyres checked.

Q40. Two factories manufacture the same machine parts. Each part is classified as having either 0, 1, 2 or 3 manufacturing defects. The joint probability for this is given below :

Manufacturer	Number of Defects			
	0	1	2	3
X	0.1250	0.0625	0.1875	0.1250
Y	0.0625	0.0625	0.1250	0.2500

(i) A part is observed to have no defect. What is the probability that it was produced by X manufacturer ?

(ii) A part is known to have been produced by manufacturer X. What is the probability that the part has no defects ?

(iii) A part is known to have two or more defects. What is the probability that it was manufactured by X.

(iv) A part is known to have one or more defects. What is the probability that it was manufactured by Y.

Q41. The management of a company has recorded the following analysis of its employees : **[Eco. (H) 2011]**

Age	Male	Female	Total
Under 30	90	10	100
30 – 40	20	30	50
Over 40	40	10	50
Total	150	50	200

If one employee is selected at random, find :

(i) The probability that the employee is a male.

(ii) The probability that the employee is female, given that the employee is over 40.

(iii) The probability that the employee is under 30, given that he is male.

(iv) The probability that the employee is over 40 given that he is male.

Q42. An investment analyst collects data on stocks and notes, whether or not dividends were paid and whether or not the stocks increased in price over a given period. Data are presented in the following table :

	Price Increase	No Price Increase	Total
Dividends paid	34	78	112
No dividends paid	85	49	134
Total	119	127	246

(i) If a stock is randomly selected, what is the probability that it increased in price.

(ii) If a stock is randomly selected, what is the probability that it both increased in price and paid dividends?

(iii) What is the probability that a randomly selected stock neither paid dividends nor increased in price?

(iv) If a stock is known not to have paid dividends, what is the probability that it increased in price? **[Eco. (H) 2011]**

Q43. Given the following probability table of television viewing frequencies (X) and income levels (Y),

Viewing Frequency (X)	Income Levels (Y)			Total
	High Income	Middle Income	Low Income	
Regular	0.10	0.15	0.05	0.30
Occasional	0.10	0.20	0.10	0.40
Rarely	0.05	0.05	0.20	0.30
Total	0.25	0.40	0.35	1.00

(i) What is the probability that a person has a low income individual and views TV regularly?

(ii) If an individual is at low income level, what is the probability that he/she views TV regularly?

(iii) What is the probability that given an individual does not have high income, he/she rarely watches TV?

(iv) If an individual occasionally watches TV, what is the probability he/she is a high income earner or a middle income earner?

(v) Is viewing TV regularly independent of earning high income? Explain.

[Eco. (H) I Sem. 2011]

Q44. A market survey was conducted in four cities pertaining to the preference for brand X over brand Y. the responses were tabulated as below :

	Delhi	Jaipur	Agra	Jammu
Prefer X	45	55	60	50
Prefer Y	35	45	35	45
Indifferent	5	5	5	5

(i) What is the probability that a consumer selected at random preferred brand X?

(ii) What is the probability that a consumer preferred brand X and was from Agra?

(iii) Given that a consumer preferred brand X, what is the probability that he was from Agra or Jammu?

(iv) What is the probability that a consumer preferred brand X, given that he was from Agra or jammu? **[Eco. (H) I Sem. 2012]**

Q45. 200 students from standard IV, V and VI from a certain city school were surveyed and asked if they liked mathematics. Their answers are given below :

	Yes	**No**
Standard IV	52	38
Standard V	64	18
Standard VI	20	8

[Eco. (H) II Sem. 2014]

If one of the surveyed students is selected at random, what is the probability that :

(i) He belongs to standard V or he has answered Yes.

(ii) He belongs to standard IV and he has answered No.

(iii) He has answered yes given that he belongs to either standard IV or standard V.

(iv) He has answered No, given that he belongs to standard VI.

Problems Involving Proofs

Q46. Prove the following : **[Eco. (H) Stephens 2010]**

(i) If $P(A/B) \geq P(A)$, then $P(B/A) \geq P(B)$

(ii) If $P(B/A) = P(B/\overline{A})$, then A and B are independent trials.

Q47. For any events A and B with P(B) > 0, show that $P(A/B) + P(\overline{A}/B) = 1$.

MA Entrance

Q1. A dog has two puppies and a cat has two kittens. You know the older puppy is male. You also know that at least one of the kittens is male. What are the probabilities that the dog and the cat each have two male off springs?

(a) ½ and ½ respectively (b) ½ and ¼ respectively

(c) ½ and 1/3 respectively (d) ½ and 2/3 respectively

[**Ans. :** (c)] **[DSE MA Ent. Eco. 2001]**

Q2. A die is loaded in such a way that each odd number is twice as likely to occur as each even number. **[DSE MA Ent. Eco. 2002]**

(a) Find the probability that a number greater than 3 occurs on a single roll of a die.

(b) What is the probability that the number rolled is a perfect square?

(c) What is the probability that the number rolled is a perfect square given that it is greater than 3?

[**Ans. : (a)** 4/9, **(b)** 1/3, **(c)** ¼]

Q3. The depositors at Save-More Bank are categorized by age and sex. We are going to select an individual at random from this group of 2000 depositors.

Age	Sex	
	Male	Female
30 or less	800	600
31 or more	400	200

Then the probability the depositor is female = **[DSE MA Ent. Eco. 2002]**

(a) 3/10 (b) 2/5 (c) 3/5 (d) 2/3

[**Ans. :** (b)]

Q4. What is the conditional probability that the above depositor drawn is 30 or less, given that he is a male? **[DSE MA Ent. Eco. 2002]**

(a) 2/3 (b) 7/10 (c) 4/7 (d) 2/5

[**Ans. :** (a)]

Q5. Consider the experiment of tossing two unbiased coin in succession. What is the probability of obtaining two heads, given that at least one of the coins comes up heads? **[DSE MA Ent. Eco. 2006]**

(a) ½ (b) ¼ (c) 1/3 (d) 2/3

[**Ans. :** (c)]

Q6. A fair dice has numbers 1,2,3, 4, 5 and 6 on its sides. It is tossed once. I win Rs. 1 if an odd number shows up; otherwise I lose Rs.1. Let X be the number that shows up and Y the money I win. [**Note :** Y < 0 if I lose money.]

Which of the following is incorrect? **[DSE MA Ent. Eco. 2009]**

(a) Prob (X > Y) = 1 (b) Prob(X= 3 | Y = 1) = 1/3
(c) E(Y) = 0 (d) Prob(Y = 1| X =5) = 1
[**Ans. :** (a)]

Q7. You've been told that a family has two children and one of these is a daughter. What is the probability that the other child is also a daughter?
(a) ½ (b) 1/3 (c) ¼ (d) 3/4
[**Ans. :** (b)] **[DSE MA Ent. Eco. 2009]**

Q8. A number, say X_1, is chosen at random from the set {1, 2}. Then a number, say X_2, is chosen at random from the set {1, X_1}. The probability that $X_1 = 2$ given that $X_2 = 1$ is **[DSE MA Ent. Eco. 2010]**
(a) 1 (b) ½ (c) 1/3 (d) 1/4
[**Ans. :** (c)]

Q9. Suppose, you are an editor of a magazine. Every day you get two letters from your correspondents. Each letter is as likely to be from a male as from a female correspondent. The letters are delivered by a post-man, who brings one letter at a time. Moreover, he has a `ladies first' policy; he delivers letter from a female first, if there is such a letter. Suppose you have already received the first letter for today and it is from a female correspondent. What is the probability that the second letter will also be from a female? **[DSE MA Ent. Eco. 2013]**
(a) ½ (b) ¼ (c) 1/3 (d) 2/3
[**Ans. :** (a)]

Answers of Exercise 5

1.	4/9,	**2.**	3/7,	**3.**	0.5,	**4.**	1/3,	**5.(i)**	¾,
(ii)	7/12,	**(iii)**	5/12,	**6.**	3/14, 0.85	**7.(i)**	0.4,	**(ii)**	0.2,
(iii)	0.4,	**8.(i)**	0.5,	**(ii)**	0.2,	**(iii)**	0.4,	**9.**	1/3,
10.	2/15,	**11.**	1/3,	**12.**	2/5,	**13.**	3/10,	**15.**	1/15,
16.	2/9,	**17.**	1/3,	**18.**	1/6,	**19.**	3/5,	**20.**	½,
21.	½,	**22.**	5/8,	**23.**	½,	**24.**	0.12, 0.6,	**25.**	2/9,
26.	1/3,	**27.(a)**	{HH, HT, TH, TT},			**(b)**	1/3,	**28.(i)**	4/9
(ii)	5/8,	**29.**	7/10,	**30.**	1/15,	**31.**	1/3	**32.**	0.099,
35.(i)	65/100,	**(ii)**	35/100,	**(iii)**	1/5,	**(iv)**	0	**(v)**	8/13,
(vi)	8/35,	**36.(i)**	1/5,	**(ii)**	87/90,	**37.(i)**	0.616,	**(ii)**	0.625,
38.(i)	3/20,	**(ii)**	½,	**39.(i)**	0.66,	**(ii)**	0.24,	**40.(i)**	0.67,
(ii)	0.25,	**(iii)**	0.45,	**(iv)**	0.54,	**41.(i)**	¾,	**(ii)**	1/5,
(iii)	3/5,	**(iv)**	4/15,	**42.(i)**	119/246,	**(ii)**	17/123,		
(iii)	49/246,	**(iv)**	85/246,	**43.(i)**	0.05,	**(ii)**	1/7,	**(iii)**	1/3,
(iv)	¾,	**(v)**	No	**44.(i)**	21/39,	**(ii)**	2/13,	**(iii)**	11/21,
(iv)	11/20								

Basic Concepts

Multiplication theorem of Probability

1. **Multiplicative Rule :** If A and B are two events, associated with a random experiment then
   $P(A \cap B) = P(A).P(B/A)$, where $A \neq \phi$, and
   $P(A \cap B) = P(B).P(A/B)$, where $B \neq \phi$,
2. **Independent Events :** Two events A and B are said to be independent if the occurrence or non occurrence of one does not affect the probability of the occurrence or non occurrence of the other. Thus, if events A and B are independent then P(A/B) = P(A) and P(B/A) = P(B).

   But, $P(A/B) = P(A) \Rightarrow \frac{P(A \cap B)}{P(B)} = P(A) \Rightarrow P(A \cap B) = P(A).P(B)$

   Also, $P(B/A) = P(B) \Rightarrow \frac{P(A \cap B)}{P(A)} = P(B) \Rightarrow P(A \cap B) = P(A).P(B)$

Note : If A and B are independent events, then
(i) A and B' are also independent, i.e. $P(A \cap B') = P(A).P(B')$, and
(ii) A' and B are also independent, i.e. $P(A' \cap B) = P(A').P(B)$, and
(iii) A' and B' are also independent events i.e. $P(A' \cap B') = P(A').P(B')$.

Note : If A, B and C are independent events, then
$P(A \cap B \cap C) = P(A).P(B).P(C)$

Trick : If $A_1, A_2, A_3, \ldots A_n$ are n independent events associated with a random experiment, then
$P(A_1 \cup A_3 \cup A_3 \ldots \cup A_n) = 1 - P(A_1').P(A_2').P(A_3').\ldots\ldots P(A_n')$

3. **Independent Experiments :** Two random experiments are said to be independent, if for every pair of events A and B where A is associated with the first and B is associated with the second experiment, the probability of the simultaneous occurrence of A and B, when two experiments are performed, is the product of their separate probabilities.

Example 5 : A bag contains 5 black and 7 red balls. One ball is drawn from this bag and is replaced then another ball is drawn. What is the probability of drawing red balls in both drawings.

Solution : For first draw

Let S be the sample space and A be the event of drawing a red ball

$\therefore$ n(S) = 12 and n(A) = 7

Now, $P(A) = \frac{n(A)}{n(S)} = \frac{7}{12}$

For second draw

Since second draw is made after replacing the ball drawn in first draw

$\therefore$ S is also the sample space for second draw and B be the event of drawing a red ball

$\therefore$ n(S) = 12 and n(B) = 7

Now, $P(B) = \frac{n(B)}{n(S)} = \frac{7}{12}$

Since A and B are independent events

$\therefore$ $P(A \cap B) = P(A).P(B) = \frac{7}{12} \times \frac{7}{12} = \frac{49}{144}$

Exercise 6

Q1. Events A and B are such that P(A) = ½, P(B) = 7/12 and P(not A or not B) = ¼. State whether A and B are independent.

Q2. Events E and F are given to be independent. Find P(F) if it is given that P(E) = 0.60 and P(E∩F)=0.35.

Q3. If P(not B) = 0.4, P (AUB) = 0.75 and A, B are independent events, find the value of P(A).

Q4. If A and B are two independent events such that P(A') = 0.65 where A' = non happening of A, P (AUB) = 0.65 and P (B) = p, find the value of p.

Q5. If A and B are independent events such that, P(A) = ½, P(B) = $\frac{1}{3}P(A)$. Find $P(A \cup B)$.

Q6. If P(A) = 2/3, P(B) = 1/5 find $P(A \cup B)$ if
(i) A and B are mutually exclusive
(ii) A and B are independent.

Q7. It is given that $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$. Show that A and B are independent events. **[Eco. (H) 1997]**

Q8. Let C_1, C_2 and C_3 be independent events with probabilities ½, 1/3 and ¼ respectively. Compute $P(C_1 \cup C_2 \cup C_3)$. **[Eco. (H) I Sem. 2014]**

Q9. It is given that two events, A and B, are both independent and mutually exclusive. Show that at least one of them must have zero probability. **[Eco. (H) 1998]**

Q10. A coins is tossed thrice and all eight outcomes are equally likely,
E is "the first throw results in head".
F is "the last throw results in tail
Prove that events E and F are independent.

Q11. X and Y appear in an interview for two posts. The probability of X's selection is 1/7 and that of Y's selection is 1/5. What is the probability that
(i) both of them will be selected? (ii) Only one of them will be selected?
(iii) None of them will be selected? (iv) At least one of them will be selected?

Q12. A salesman has a 60% chance of selling a product to a customer. The behaviour of successive customers is independent. If two customers A and B enter the shop, what is the probability that the salesman will make a sale to A or B? **[Eco (H) 2000]**

Q13. The probability that a teacher will give an unannounced test during any class meeting is 1/5. If a student is absent twice, what is the probability that he will miss at least one test? **[Eco (H) 2003]**

Q14. A doctor sees 4 patients in a day. Events A and B are defined as follows :
A : {All patients are of same sex}
B : {At least 3 patients are men}
(i) What elements of sample space constitute the events A and B?
(ii) Are A and B independent?
(iii) If the doctor chooses to see 3 patients instead of 4, events A and B remain unchanged. Will A and B still be independent? **[Eco. (H) II Sem. 2014]**

Q15. A card is drawn from each of the two well shuffled packs of cards. Find the probability that at least one of them is an ace. **[Eco. (H) II Sem. 2014]**

Q16. The probabilities that a student receives A grade in three courses are 0.2, 0.3 and 0.9 respectively. If the grades can be regarded as independent events, find that he receives (i) All A's (ii) No A's (iii) exactly two A's

Q17. A scientist has to make a decision on each of the two independent events I and II. Suppose the probability of error in making a decision on event I is 0.02 and that on event II is 0.05. Find the probability that the scientist will make the correct decision on (i) both events, (ii) only one event.

Q18. A machine operates if all of its three components function. The probability that the first component fails during the year is 0.14, the second component fails is 0.10 and the third component fails is 0.05. What is the probability that the machine will fail during the year ?

Q19. The results of an investigation by an expert on a fire accident are summarized below :
(i) Probability there could have been short circuit = 0.8
(ii) Probability LPG explosion cylinder = 0.2
(iii) Chance of fire accident is 30% given a short circuit and 95% given an LPG explosion.
Based on these, what do you think is the most probable cause of fire.

Q20. A problem in statistics is given to two students A and B. The odds in favour of A solving the problem are 6 to 9 and against B solving the problem are 12 to 10. if both A and B attempt, find the probability of the problem being solved.

Q21. The odds against A solving a problem are 10 to 7 and the odds in favour of B solving the problem are 15 to 12. What is the probability that if both of them try the problem will be solved?

Q22. Three ships A, B and C sail from England to India. Odds in favour of their arriving safety are 2:5, 3:7 and 6:11. Find the probability that they all arrive safely. **[Eco. (H) 2000]**

Q23. A speaks truth in 60% of the cases and B in 90% of the cases. What percentage of cases are they likely to contradict each other in stating the same fact.

Q24. A speaks truth in 60 percent cases and B speaks truth in 70 percent cases. In what percentage of cases are they, likely to contradict each other in stating the same fact.

Q25. In a family, the husband tells a lie in 30% cases and the wife in 35% cases. Find the probability that both contradict each other on the same fact.

Q26. A and B appear for an interview for two parts. The probability of A's selection is 1/3 and that of B's selection is 2/5. Find the probability that only one of them will be selected.

Q27. The probability that a student A can solve a question is 6/7 and that another student B solving a question is 3/4. Assuming that two events "A can solve the question" and "B can solve the question" are independent, find the probability that only one of them solves the question.

Q28. The probabilities of A, B, C solving a problem are 1/3, 2/7 and 3/8 respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

Q29. A problem in statistics is given to three students A, B and C whom chances of solving it are 1/2,1/3 and 1/4 respectively. Find the probability that only one of them solves it correctly.

Q30. The probability of student A passing an examination is 2/9 and of student B passing is 5/9. Assuming the two events: 'A passes' & 'B passes' as independent find the probability of :
(i) Only A Passing the examination.
(ii) Only one of them passing the examination.

Q31. A, B and C can hit a target 4 times in 5 shots; 3 times in 4 shots and 2 times in 3 shots respectively. Find the probability that exactly two of them will hit the target.

Q32. Three marksmen can hit the target with probabilities ½, 1/3 and ¼ respectively. They shoot simultaneously. What is the probability that there are two hits?

Q33. Three marksman hit the target with probabilities ½, 1/3 and ¼ respectively. They shoot simultaneously and there are two hits. Who missed? Find the probabilities. **[Eco. (H) 2002]**

Q34. The odds are 9 to 5 against a person aged 50 years living till he is 70 and 8 to 6 against a person aged 60 years living till he is 80. Find the probability that at least one of them will be alive after 20 years. **[Eco. (H) 1996]**

Q35. A husband and wife appear in an interview for two vacancies for the same post. The probability of husband's selection is 1/7 and that of wife's selection is 1/5. What is the probability that at least one of them will be selected.

Q36. Ramesh appears for an interview for two posts A and B for which selection is independent. The probability for his selection for post A is 1/6 and for post B is 1/7. Find the probability that Ramesh is selected for at least one of the posts.

Q37. The probability that A can solve a problem is ¼ and that B can solve it is 1/3. If both of them try, what is the probability that the problem will be solved.

Q38. A can solve 90% of the problems given in a book and B can solve only 70% problems. What is the probability that at least one of them will solve the problems selected at random from the book ?

Q39. A problem in mathematics is given to three students whose chances of solving it are 1/2, 1/3, 1/4. What is the probability that the problem will be solved.

Q40. Four tanks are shooting at an enemy aircraft with their respective probabilities of hitting the target being 0.1, 0.2, 0.3 and 0.4. Find the probability that the aircraft is shot down.

Q41. Suppose 10% of the students fail in an examination. If 3 students are selected at random, find the probability that at most 1 student will fail.

Q42. An unbiased die is tossed 3 times. If getting a '2' or '5' is considered a success. Find the probability of at least two successes.

Q43. To test the quality of electric bulbs produced in a factory, two bulbs are randomly selected from a large sample without replacement. If either bulb is defective, the entire lot is rejected. Suppose a sample of 200 bulbs contains 5 defective bulbs. Find the probability that the sample will be rejected.

Q44. A doctor claims that 60% of the patients he examines are allergic to some type of weed. What is the probability that
(i) exactly 3 of his next 4 patients are allergic to weeds ?
(ii) none of his next 4 patients is allergic to weeds ?

Q45. In a village of 21 inhabitants, a person tells a rumour to a second person, who in turn repeats it to a third person, etc. At each step the receipent of the rumour is chosen at random from the 20 people available. Find the probability that the rumour will be told 10 times without:
(i) returning to the originator;
(ii) being repeated to any person; **[Eco. (H) 2002]**

Q46. A can hit a target 4 times in 5 shots. B three times in 4 shots and C two times in three shots. They fire a volley, what is the probability that the target is damaged, if at least two shots are required to damage it. **[Eco. (H) 2006]**

Q47. A telemarketer calls people and tries to sell them a subscription to a daily newspaper. On 25% of her calls, there is no answer or the line is busy. She sells a subscription to 10% of the remaining calls. For what proportion of calls does she make a sale? **[Eco. (H) I Sem. 2012]**

Q48. If A and B are independent events does it follow that A' (complement of A) and B' (complement of B) are also independent? **[Eco. (H) I Sem. 2012]**

Q49. A string of Diwali lights has 5 bulbs wired in a series, *i.e.*, the string will work only if all the bulbs work. If each bulb has 90% chance of working the first time the current is applied, what is the probability that the string will not work? Assume bulb failures are independent events. **[Eco. (H) I Sem. 2012]**

Q50. A recent survey revealed that one out of 10 persons in a certain town has high blood pressure. If 12 persons in this town are randomly chosen and tested, what is the probability that at least one of them will have high blood pressure? **[Eco. (H) II Sem. 2014]**

Drawing items with replacement

Q51. Tickets are numbered 1 to 10. Two tickets are drawn one after the other at random. Find the probability that the number on one of the tickets is a multiple of 5 and another a multiple of 4.

Q52. Cards are numbered 1 to 25. Two cards are drawn one after the other. Find the probability that the number on one card is a multiple of 7 & on the other it is a multiple of 11.

Q53. A bag contains 8 marbles of which 3 are blue and 5 are red. One marble is drawn at random, its colour is noted and the marble is replaced in the bag. A marble is again drawn from the bag and its colour is noted. Find the probability that the marbles will be

(i) blue followed by red, (ii) blue and red in any order

(iii) of the same colour.

Q54. A bag contains 4 white and 3 black balls. Four balls are successively drawn out with replacement. Find the probability that they are alternatively of different colours.

Q55. A bag contains 5 white, 7 red and 4 black balls. If four balls are drawn one by one with replacement. What is the probability that none is white.

Q56. A bag contains 7 green , 4 white and 5 red balls. If four balls are drawn one by one with replacement, what is the probability that none is red.

Q57. A bag contains 8 red, 7 green and 5 black balls. If four balls are drawn one by one with replacement. Find the probability that none is a black ball.

Q58. A bag contains 5 white , 7 red and 8 black balls. If four balls are drawn one by one with replacement, what is the probability that at least one is white.

Drawing items without replacement

Q59. A bag contains 5 white and 3 black balls. Four balls are successively drawn out without replacement. What is the probability that they are alternatively of different colours.

Q60. A bag contains 5 white, 7 red and 8 black balls. If four balls are drawn one by one without replacement. What is the probability that all are white.

Q61. A box contains 3 red and 5 blue balls. Two balls are drawn at random without replacement. Find the probability of getting 1 red and 1 blue ball.

Q62. A bag contains 3 white and 5 black balls. Two balls are drawn at random without replacement. Determine the probability of getting both the balls black.

Q63. Two balls are drawn at random from a bag containing 2 white, 3 red, 5 green and 4 black one by one without replacement. Find the probability the both the balls are of different colours.

Q64. Two cards are drawn from a well shuffled pack of 52 cards, one after another without replacement. Find the probability that one of these is an ace and the other is a queen of the opposite shade.

Q65. Two cards are drawn from a well shuffled pack of 52 cards without replacement. Find the probability that neither Jack nor a Card of spade is drawn.

Q66. Two cards are drawn without replacement from a well shuffled pack at 52 cards. Find the probability that one is a spade and other is a queen of red colour.

Q67. Two cards are drawn from a well shuffled pack of 52 cards without replacement. What is the probability that one is a red queen and other is a king of black colour.

Q68. A bag contains 5 white and 3 black balls. Two balls are drawn at random one after the other without replacement, find the probability that balls drawn are black.

[Eco. (H) 1993]

Q69. A box of nine golf gloves contains two left-handed and seven right-handed gloves:
(i) If two gloves are randomly selected from the box without replacement, what is the probability that –(x) both gloves are right handed and (y) one left-handed and one right-handed glove?
(ii) If three gloves are selected without replacement, what is the probability that all of them are left-handed?
(iii) If two gloves are randomly selected with replacement, what is the probability that they would both be right-handed?

Q70. Bag I and Bag II each have 3 blue and 3 green balls. Two balls are drawn without replacement from each bag. What is the probability that exactly one of the four balls drawn is blue? **[Eco. (H) I Sem. 2014]**

Q71. Two cards are drawn from a well shuffled pack of 52 cards one after another without replacement. Find the probability that one of these is a queen and other is a king of opposite colour.

Drawing items from two or more bags(groups)

Q72. A bag contains 3 red and 5 black balls and a second bag contains 6 red and 4 black balls. A ball in drawn from each bag. Find the probability that one is red and other is black.

Q73. A bag contains 4 white balls and 2 black balls. Another bag contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability that both are black.

Q74. A bag contains 6 black and 3 white balls. Another bag contains 5 black and 4 white balls. If one ball is drawn from each bag, find the probability that, these two balls are of the same colour.

Q75. A bag contains 4 white balls and 2 black balls. Another contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability:
(i) Both are white (ii) Both are black (iii) One is white and one is black

Q76. Three group of children contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 2 boys. One child is selected at random from each group. Show that the probability that the three selected consist of 1 girl and 2 boys is 3/8.

Q77. There are three Urns A, B and C. Urn A contains 4 red balls and 4 green balls. Urn B contains 3 red balls and 5 green balls. Urn C contains 5 red and 2 green balls. One ball is drawn from each of the three urns. What is the probability that out of these three balls drawn two are green balls and one is a red ball.

Q78. Three bags contain 7 white & 8 red, 9 white & 6 red, and 5 white & 7 red balls respectively. One ball, at random is drawn from each bag. Find the probability that all of them are of same colour.

Q79. A bag has 4 red and 5 black balls, a second bag has 3 red and 7 black balls. One ball is drawn from the first and two from the second. Find the probability that, out of three balls, two are black and one is red.

Q80. The bag A contains 5 red and 3 green balls and bag B contains 3 red and 5 green balls. One ball is drawn from bag A and two from bag B, find the probability that, of the three balls drawn, two are red and one is green.

Q81. Bag A contains 5 red and 3 green balls and bag B contains 3 red and 5 green balls. One ball is drawn from bag A and two from bag B, find the probability that all the balls are green.

Transferring items from one group(bag) to another group(bag)

Q82. A bag contains 3 white and 5 black balls, and a second bag contains 5 white and 3 black balls. One ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.

Q83. A bag contains 4 red and 7 green balls and another bag contains 8 white and 5 green balls. One ball is transferred from the first bag to the second bag without seeing its colour. A ball is then drawn from the second bag. Find the probability that ball drawn is green in colour.

Repeated Trials

Q84. A and B toss a coin alternately till one of them tosses a head and wins the game. If A starts the game find their respective probabilities of winning.

Q85. A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning if A starts first.

Q86. A and B play with a pair of dice, one who throws greater than 9 wins the game. If A starts first, find their chances to win the game.

Q87. A and B throw a pair of dice alternatively beginning with A. Find the probability that A gets a total of 9 before B gets a doublet, and wins.

Q88. A and B throw alternatively with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. A begins the game. Show that the odds in favour of A are 30 : 31.

Q89. Three persons A, B and C throw a die in succession till one gets a six and wins the game. If A starts first, followed by B and then C. Find their respective probabilities of winning.

Q90. A, B and C, in order play a game and chances of their winning it in an attempt are 2/3, ½ and ¼ respectively. A has first chance followed by B and then by C. this cycle is repeated till one of them wins the game. Find their respective chances of winning.

Q91. An urn contains ten balls of which six are black and four are white. The following game is played. At each trial a ball is selected at random, its colour noted, and it is replaced along with two additional balls of the same colour. What is the probability a white ball is selected in each of the first three trials? **[Eco. (H) I Sem. 2011]**

Problems Involving Proofs

Q92. Prove that two mutually exclusive events with positive probabilities cannot be independent. **[Eco. (H) 2008, Eco. (H) Deshbandhu 2010]**

Q93. It is given that two events A & B are both independent and mutually exclusive. Show that at least one of them must have zero probability. **[Eco. (H) HRC 2009]**

Q94. When will the events A and B be both independent and mutually exclusive. **[Eco. (H) Kalindi 2010]**

MA Entrance

Q1. Given P(A) = 0.5 and P(B) = 0.4, if A and B are independent the $P(A \cup B)$ is
(a) 0.2 (b) 0.7 (c) 0.9 (d) None of the above
[**Ans. :** (b)] **[DSE MA Ent. Eco. 2000]**

Q2. Three players A, B and C take turns playing a game as follows. A and B play in the first round. The winner plays C in the second round, while the loser sits out. The winner of the second round plays the person who was sitting out. The game continues in this fashion, with the winner of the current round playing the next round with the person who sits out in the current round. The game ends when a player wins twice in succession; this player is declared the winner of the contest. For any of the rounds, assume that the two players playing the round each have a probability ½ of winning the round, regardless of how the past rounds were won or lost. **[DSE MA Ent. Eco. 2006]**

(i) The probability that A becomes the winner of the contest is
(a) 5/14 (b) ½ (c) 3/7 (d) 7/16
[**Ans. :** (a)]

(ii) The probability that C becomes the winner of the contest is
(a) 1/7 (b) 1/5 (c) 1/8 (d) 2/7
[**Ans. :** (d)]

(iii) The probability that the game continues indefinitely, with no one winning twice in succession, is
(a) $1/10^{23}$ (b) 0 (c) $1/2^{23}$ (d) $1/2^{16}$
[**Ans. :** (b)]

Q3. Amit has box containing 6 red balls and 3 green balls. Amita has a box containing 4 red balls and 5 green balls. Amit randomly draws one ball from his box and puts it into Amita's box. Now Amita randomly draws one ball out of her box. What is the probability that the balls drawn by Amit and Amita were of different colours?
(a) 1/3 (b) 2/15 (c) 4/17 (d) 7/15
[**Ans. :** (d)] **[DSE MA Ent. Eco. 2006]**

Q4. ICICI bank collects data on 10000 respondents. Out of the 6800 men, 4200 have credit cards, and out of the 3200 women, 2500 have credit cards. Out of the men with credit cards, 1200 have unpaid balances, whereas out of the women with credit cards, 1400 have unpaid balances. What is the probability that an individual selected at random is a man without an unpaid balance? **[DSE MA Ent. Eco. 2006]**
(a) 0.68 (b) 0.56 (c) 0.12 (d) 0.84
[**Ans. :** (b)]

Q5. There are 3 red and 5 black balls in an urn. You draw two balls in succession without replacing the first ball. What is the probability that the second ball you draw is red? **[DSE MA Ent. Eco. 2006]**
(a) 2/7 (b) 3/8 (c) 5/7 (d) 1/4
[**Ans. :** (b)]

Q6. A traffic light on the way to the university is red 40% of the time. What's the probability of getting a red light (i) 2 days in a row, (ii) any 2 out of 3 days.
(a) (i) 0.16; (ii) 0.20 (b) (i) 0.16; (ii) 0.29

(c) (i) 0.24; (ii) 0.29 (d) none of the above

[**Ans. :** (b)] **[DSE MA Ent. Eco. 2007]**

Q7. Let A and B be any two events, each of which has a positive probability of occurring. Consider the following statements **[DSE MA Ent. Eco. 2008]**

I If A and B are independent, they must be mutually exclusive

II If A and B are mutually exclusive, they must be independent

III If A and B are independent, they cannot be mutually exclusive

IV If A and B are mutually exclusive, they cannot be independent

Which of the above statements are true?

(a) I and IV (b) II and III

(c) III and IV (d) None of the statements are true

[**Ans. :** (c)]

Q8. Suppose that the probability that any particle emitted by a radioactive material will penetrate a given shield is 0.01. If ten particles are emitted, what is the probability that exactly one of the particles will penetrate the shield?

(a) $(0.01)(0.99)^9$ (b) $(0.1)(0.99)^9$ (c) 0.1 (d) 1/9

[**Ans. :** (a)] **[DSE MA Ent. Eco. 2008]**

Q9. A company has 100 employees, 40 men and 60 women. There are 6 male executives. How many female executives should there be for gender and rank to be independent? **[DSE MA Ent. Eco. 2009]**

(a) 9 (b) 6 (c) 10 (d) 8

[**Ans. :** (a)]

Q10. Consider two events A and B with Pr(A) = 0.4 and Pr(B) = 0.7 The maximum and minimum values of Pr(A∩B) respectively are : **[DSE MA Ent. Eco. 2009]**

(a) (0.4; 0.1) (b) (0.7; 0.4) (c) (0.7; 0.1) (d) (0.4; 0)

[**Ans. :** (a)]

Q11. Two persons, A and B, shoot a target. Suppose the probability that A will hit the target on any shot is 1/3 and the probability that B will hit the target on any shot is ¼. Suppose A shoots first and they take turns shooting. What is the probability that the target is hit for the first time by A's third shot? **[DSE MA Ent. Eco. 2010]**

(a) 1/24 (b) 1/12 (c) 1/6 (d) 1/3

[**Ans. :** (b)]

Q12. In order to join the 'Gamers Club', Mr A must choose a box from the two identical boxes in a room, and draw one ball from the chosen box. All he knows is that both boxes are non empty, and have a mix of red and green balls. If the ball that he draws from his chosen box is green, he is admitted to the club. You are given two identical boxes, 50 red balls and 50 green balls and asked to allocate these balls to the two boxes in order to maximize in order to maximize Mr. A's probability of being admitted to the club, given that he will choose a box randomly. If you allocate these balls correctly, the probability that Mr. A gets admitted to the club equals **[DSE MA Ent. Eco. 2011]**

(a) 1 (b) 0.747 (c) 0.547 (d) 0.257

[**Ans. :** (b)]

Q13. An urn contains equal number of green and red balls. Suppose you are playing the following game. You draw one ball at random from the urn and note its colour. The ball is then placed back in the urn, and the selection process is repeated. Each time a green ball is picked you get 1 Rupee. The first time you pick a red ball, you pay 1 Rupee and the game ends. Your expected income from this game is
(a) 1 (b) Positive but finite (c) Zero (d) Negative
[**Ans. :** (c)] **[DSE MA Ent. Eco. 2012]**

Q14. A fair coin is tossed until a head comes up for the first time. The probability of this happening on an odd-numbered toss is **[DSE MA Ent. Eco. 2012]**
(a) ½ (b) 1/3 (c) 2/3 (d) ¾
[**Ans. :** (c)]

Q15. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, then the number of outcomes B must have so that A and B are independent, is
(a) 4 (b) 3 or 9 (c) 6 (d) 5 or 10
[**Ans. :** (d)] **[DSE MA Ent. Eco. 2012]**

Q16. A particular men's competition has an unlimited number of rounds. In each round, every participant has to complete a task. The probability of a participant completing the task in a round is p. If a participant fails to complete the task in a round, he is eliminated from the competition. He participates in every round before being eliminated. The competition begins with three participants. The probability that all three participants are eliminated in the same round is :
(a) $\frac{(1-p)^3}{1-p^3}$ (b) $\frac{1}{3}(1-p)$ (c) $\frac{1}{p^3}$ (d) None of the above
[**Ans. :** (a)] **[ISI MS(QE) 2016]**

Answers of Exercise 6

1.	No,	**2.**	7/12,	**3.**	3/8,	**4.**	6/13,	**6(i).**	13/15,
(ii)	11/15,	**11(i)**	1/35,	**(ii)**	2/7,	**(iii)**	24/35,	**(iv)**	11/35,
12.	0.84,	**13.**	0.36,	**16.(i)**	0.054,	**(ii)**	0.056	**(iii)**	0.348,
17.(i)	0.931,	**(ii)**	0.068,	**18.**	0.26,	**19.**	short circuit,	**20.**	18/25,
21.	113/153,	**22.**	18/595,	**23.**	42/100,	**24.**	46%,	**25.**	44%,
26.	7/15,	**27.**	9/28,	**28.**	25/56,	**29.**	11/24,	**30(i).**	8/81,
(ii)	43,81,	**31.**	13/30,	**32.**	¼,	**33.**	¼,	**34.**	0.633,
35.	11/35,	**36.**	2/7,	**37.**	1/2,	**38.**	97/100,	**39.**	3/4,
40.	0.6976,	**41.**	$\frac{6}{5}\left[\frac{9}{10}\right]^2$,	**42.**	7/27,	**43.**	0.0495,	**44.(i)**	$\frac{216}{625}$
(ii)	16/625,	**45.(i)**	$\frac{1}{20}\times\left(\frac{1}{19}\right)^9$,	**(ii)**	$\left(\frac{1}{20}\right)^{10}$,	**46.**	5/6,	**47.**	7.5%,
49.	0.40951,	**51.**	4/45,	**52.**	6/625,	**53(i)**	15/64,	**(ii)**	15/32,
(iii)	17/32,	**54.**	6/35,	**55.**	$\left[\frac{11}{16}\right]^4$,	**56.**	$\left[\frac{11}{16}\right]^4$,		
57.	81/256,	**58.**	175/256,	**59.**	1/7,	**60.**	1/969,	**61.**	15/28,

62. 5/14, **63.** 71/91, **64.** 4/663, **65.** 105/221, **66.** 1/51,
67. 2/663, **68.** 3/14, **69.(i)(x)** 7/12, **(y)** 7/18, **(ii)** 0, **(iii)** 49/81
71. 4/663, **72.** 21/40, **73.** 5/24, **74.** 14/27, **75(i)** 1/4,
(ii) 5/24, **(iii)** 13/24, **77.** 41/112, **78.** 217/900, **79.** 7/15,
80. 3/8, **81.** 15/112, **82.** 43/72, **83.** 31/77, **84.** 1/3,
85. P(A wins) = 6/11, P(B wins) = 5/11,
86. P(A wins) = 6/11, P(B wins) = 5/11, **87.** P(A wins) = 3/7, P(B wins) = 4/7,
89. P(A wins) = 36/91, P(B wins) = 30/91, P(C wins) = 25/91,
90. P(A wins) = 16/21, P(B wins) = 4/21, P(C wins) = 1/21, **91.** 4/35,

Basic Concepts

Baye's Theorem

1. **Law of Total probability :** Let S be the sample space and let H_1, H_2, H_3, …., H_n be n mutually exclusive and exhaustive events associated with a random experiment. If E is any event which occurs with H_1, H_2, H_3, …., H_n, then
$$P(E) = P(H_1)P(E/H_1) + P(H_2)P(E/H_2) + ... + P(H_n)P(E/H_n)$$

2. **Baye's Theorem :** If H_1, H_2, H_3, ….., H_n be n mutually exclusive and exhaustive events associated with a random experiment and E is an event which occurs together with either of H_1, H_2, H_3, ….., H_n then
$$P(H_k/E) = \frac{P(H_k).P(E/H_k)}{\sum_{i=1}^{n}\left(P(H_i).P(E/H_i)\right)}$$

Note : If $P(H_1) = P(H_2) = P(H_3) = ... = P(H_n)$ then by Baye's theorem we have
$$P(H_k/E) = \frac{P(E/H_k)}{\sum_{i=1}^{n} P(E/H_i)}.$$

Note : The events H_1, H_2, H_3, …., H_n are generally referred to as hypothesis, the probabilities $P(H_1)$, $P(H_2)$, $P(H_3)$, …., $P(H_n)$ which are known before the experiment takes place are called priori probabilities and $P(H_i/E)$ are called posteriori probabilities because they are determined after the results of the experiment are known.

Example 7 : In a bolt factory, machines A, B, C manufacture 25%, 35% and 40% respectively of the total bolts manufactured. Of their output, 5% and 4% and 2% are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, what is the probability that it is manufactured by the machine C ?

Solution : Let H_1 : the event that the bolt is manufactured by machine A
H_2 : the event that the bolt is manufactured by machine B
H_3 : the event that the bolt is manufactured by machine C
E : the event that the bolt is defective.
Now, $P(H_1) = \frac{25}{100}$, $P(H_2) = \frac{35}{100}$, $P(H_3) = \frac{40}{100}$

and $P(E/H_1)$ = Probability that bolt is defective when it is produced by machine A $= \frac{5}{100}$,

Similarly, $P(E/H_2) = \frac{4}{100}$, $P(E/H_3) = \frac{2}{100}$

Now, Required probability = Probability that the bolt is produced by machine C when it is known that the bolt is defective

$$= P(H_3/E) = \frac{P(H_3).P(E/H_3)}{P(H_1).P(E/H_1)+P(H_2).P(E/H_2)+P(H_3).P(E/H_3)}$$

$$= \frac{\frac{40}{100}\cdot\frac{2}{100}}{\frac{25}{100}\cdot\frac{5}{100}+\frac{35}{100}\cdot\frac{4}{100}+\frac{40}{100}\cdot\frac{2}{100}} = \frac{\frac{80}{100}}{\frac{125}{100}+\frac{140}{100}+\frac{80}{100}} = \frac{\frac{80}{100}}{\frac{345}{100}} = \frac{16}{69}$$

Example 8 : In a test, an examinee either guesses or copies or knows the answer to a multiple-choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct, given that he copied it is 1/8. Find the probability that he knew the answer to question, given that he correctly answered it.

Solution : Let H_1 : the event that the examinee knows the answer

H_2 : the event that the examinee guesses the answer

H_3 : the event that the examinee copies the answer

E : the event that the answer is correct.

Now, $P(H_2) = \frac{1}{3}$, $P(H_3) = \frac{1}{6}$, $P(H_1) = 1 - \{P(H_2) + P(H_3)\} = 1 - \left\{\frac{1}{3} + \frac{1}{6}\right\}$

$\Rightarrow P(H_1) = \frac{1}{2}$

and $P(E/H_1)$ = Probability that examinee gives the correct answer given that he knows the answer $= 1$

Similarly, $P(E/H_2) = \frac{1}{4}$ and $P(E/H_3) = \frac{1}{8}$

Now, Required probability

$$= P(H_1/E) = \frac{P(H_1).P(E/H_1)}{P(H_1).P(E/H_1)+P(H_2).P(E/H_2)+P(H_3).P(E/H_3)}$$

$$= \frac{\frac{1}{2}.1}{\frac{1}{2}.1+\frac{1}{3}\cdot\frac{1}{4}+\frac{1}{6}\cdot\frac{1}{8}} = \frac{\frac{1}{2}}{\frac{29}{48}} = \frac{24}{29}$$

Exercise 7

The Law of Total Probability :

Q1. There are two bags A and B containing respectively 2 white, 3 black and 3 white, 2 black balls. One bag is selected at random and one ball is drawn from it. Find the probability that it is white.

Q2. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coins is pulled at random from one of the two purses. What is the probability that it is a silver coin.

Q3. Find the probability of drawing one rupee coin from a purse with two compartments one of which contains 3 fifty paisa coins and 2 one rupees coins and the other contains 2 fifty paisa coins and 3 one rupee coins.

Q4. A firm rents bikes from 3 agencies D, E and F in the following proportions : 20% from agency D, 20% from agency E, 60% from agency F. If 10% of the bikes from agency D, 12% from agency E and 4% from agency F have bad tyres, what is the probability that the firm will get a bike with a bad tyre. **[Eco. (H) I Sem. 2012]**

Q5. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4, …., 12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8.

Q6. A manufacturer of bicycles produces 80 percent of its output in its Faridabad factory and the rest in Rewari. If the probability of a defect rate is 2% in Rewari,
(i) What is the probability a defective product was manufactured in Rewari.
(ii) If the firm replaces all defective bicycles, how many replacements will be required out of the weekly sales of 2000 bicycles? **[Eco. (H) 2010]**

Baye's Theorem :

Q7. There are two bags I and II, containing 3 red and 4 white balls, and 2 red and 3 white balls respectively. A bag is selected at random and a ball is drawn from it. If it is found to be a red ball, find the probability that it is drawn from the first bag.

Q8. Bag I contains 2 white and 3 red balls and Bag II contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn form Bag II.

Q9. There are 2 identical boxes containing respectively 4 white and 3 red balls, 3 white balls and 7 red balls. A box is choosen at random and a ball is drawn from it. Find the probability that the ball is white. If the ball is white, what is the probability that it is from the first box? **[Eco. (H) 1995]**

Q10. Three urns A, B and C contain 6 red and 4 white; 2 red and 6 white; and 1 red and 5 white balls respectively. An urn is chosen at random and a ball is drawn. If the ball drawn is found to be red, find the probability that the ball was drawn from urn A.

Q11. Three urns A, B and C contain 4 red and 6 white; 3 red and 5 white; and 2 red and 4 white balls respectively. An urn is chosen at random and a ball is drawn. If the ball drawn is found to be red, find the probability that the ball was drawn from urn A.

Q12. The contents of three urns are as follows:
Urn I : 1 white, 2 black and 3 red balls
Urn II: 2 white, 1 black and 1 red balls and
Urn III : 4 white, 5 black and 3 red balls
One urn is chosen at random and two balls are drawn. These happen to be white and red. What is the probability that they come from urn I?

Q13. There are three urns having the following compositions of black and white balls:
Urn I : 7 white and 3 black balls
Urn II : 4 white and 6 black balls
Urn III : 2 white and 8 black balls
One of these urns is chosen at random with probabilities 0.2, 0.6 and 0.2 respectively. From the chosen urn, two balls are drawn at random without

replacement. Both the balls happen to be white. Calculate the probability that the balls drawn were from Urn III.

Q14. A Scooter manufacturing factory has two plants. Plant 1 manufactures 70% of Scooters and plant 2 manufactures 30%, At plant 1, 80% of scooters are rated of standard quality and at plant 2, 90% are rated of standard Quality. A scooter is picked up at random and is found to be standard quality, what is the probability that it has come from plant 2?

Q15. A company has two plants to manufacture cars. Plant A manufactures 70% of Cars and plant B manufactures 30%, At plant A, 80% of cars are of standard quality and at plant B, 90% cars produced are of standard Quality. A car is picked up at random and is found to be of standard quality. What is the chance that it has come from plant B?

Q16. A company has two plants to manufacture bicycles. The first Plant manufactures 60% of the bicycles and the second plant 40%. 80% of bicycles are rated of standard quality at the first plant and 90% are rated of standard Quality at second plant. A bicycle is picked up at random and is found to be standard quality, what is the probability that it has come from second plant?

Q17. A company has three plants to manufacture 8,000 scooters in a month. Out of 8,000 scooters, Plant I manufactures 4,000 scooters. Plant II manufactures 3,000 scooters and Plant III manufactures 1,000 scooters. At Plant I, 85 out of 100 scooters are rated of standard quality or better, at plant II only 65 out of 100 scooters are rated of standard quality or better and at Plant III, 60 out of 100 scooters are rated of standard quality or better. What is the probability that the scooter selected at random came from (i) Plant I, (ii) Plant II and (iii) Plant III if it is known that the scooter is of a standard quality?

Q18. A company has two plants to manufacture T.V.s. The first Plant manufactures 60% of the T.V.s and the rest are manufactured by the other plant. 80% of T.V.s manufactured by the first plant are rated of standard quality while that of second plant only 70% are of standard Quality. If a T.V. is chosen at random is found to be of standard quality, what is the probability that it has come from first plant?

Q19. A factory has machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced machine A were defective and 1% produced by machine B were defective. If a defective item is drawn at random, what is the probability that is was produced by machine A?

Q20. In a bolt factory, machines A, B, C manufacture 25%, 35% and 40% respectively of the total bolts manufactured. Of their output, 5% and 4% and 2% are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, what is the probability that :

(i) it is produced by machine B

(ii) it is produced by machine B or C.

Q21. In a bolt factory, machines A, B, C manufacture 25%, 35% and 40% respectively of the total bolts manufactured. Of their output, 5% and 4% and 2% are respectively

defective bolts. A bolt is drawn at random and is found to be defective, what is the probability that it is manufactured by the either machine A or C.

Q22. The compressors used in refrigerators are manufactured by three different factories at Pune, Nasik and Nagpur. It is known that Pune factory produces twice as many compressors as the Nasik one, which produces the same number as Nagpur one (during the same period). Experience also shows that 0.2% of the compressors produced at Pune as well as at Nasik are defective and so are 0.4% of those produced at Nagpur. A quality controller chooses a compressor and finds it a defective one. What is the probability that it was produced at Nasik factory.

Q23. A factory has three machines X, Y and Z producing 1000, 2000 and 3000 bolts per day respectively. The machine X produces 1% defective bolts, Y produces 1.5% and Z produces 2% defective bolts. At the end of a day, a bolt is drawn at random and is found defective. What is the probability that this defective bolt has been produced by machine X?

Q24. A factory has three plants A, B and C. Their daily production is 500, 1000 and 2000 units. Out of these 0.5%, 0.8% and 1% units respectively are found to be defective. An item is chosen at random and is found to be defective. What is the probability that it came from plant A?

Q25. A factory has three machines A, B and C, which produce 100, 200 and 300 items of a particular type daily. Out of these 2%, 3% and 5% units respectively are found to be defective. An item is chosen at random and is found to be defective. What is the probability that it was produced by machine A?

Q26. A manufacturer uses two machines to manufacture chocolates. After a large batch has been produced, it is discovered that one of the machines, which produces 40% of total output, has a fault. As a result, 10% of the chocolates produced by the faulty machine have become impure. If a randomly selected chocolate is found to be pure, what is the probability that it was produced by the faulty machine?

[Eco. (H) II Sem. 2014]

Q27. An economist believes that during periods of high economic growth, the rupee appreciates with probability 0.7, in period of moderate economic growth the rupee appreciates with probability 0.4, and during periods of low economic growth, the rupee appreciates with probability 0.2. During any period of time, the period of high economic growth is 0.3, the probability of moderate growth is 0.5, and the probability of low economic growth is 0.2. Suppose the rupee has been appreciating during the present period, what is the probability that the country is experiencing a period of moderate economic growth?

Q28. A doctor is to visit a patient. From that past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively 3/10, 1/5, 1/10 and 2/5. the probabilities that he will be late are ¼, 1/3 and 1/12, if he comes by train, bus and scooter respectively, but if he comes by other means of transport then he will not be late, When he arrives, he is late. What is the probability that he comes by train?

Q29. Two groups are competing for the positions on the Board of Directors of a Corporation. The probabilities that the first and the second groups will win are 0.6

and 0.4 respectively. Further if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3, if the second group wins. Find the probability that the new product introduced was by the second group.

Q30. In a class 5% of the boys and 10% of the girls have an IQ of more than 150. In this class 60% of the students are boys. If a student is selected at random and found to have an IQ more than 150. Find the probability that the student is boy.

Q31. Suppose 5 men out of 100 and 25 women out of 1000 are good orators. An orator is chosen at random. Find the probability that a male person is selected. Assume that there are equal number of men and woman.

Q32. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck driver. The probability of accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets an accident. What is the probability that he is a scooter driver?

Q33. An insurance company insured 3000 scooter drivers, 4000 car drivers and 5000 truck driver. The probability of an accident involving a scooter, a car and a truck are 0.02, 0.03 and .04 respectively. One of the insured vehicles meets with an accident. What is the probability that (i) it is a scooter, (ii) it is a car, (iii) it is a truck.

Q34. An Insurance Company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The Probability of accident is 0.1, 0.3 and 0.2 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? **[Eco. (H) 2001]**

Q35. It is known from experience that 60% of all labour disputes are over wages, 15% over working conditions and 25% over other issues. Also, 45% of disputes over wages are resolved without strikes, 70% of disputes over working conditions are resolved without strikes and 40% of disputes over other issues are resolved without strikes. What is the probability that a labour management issue will be resolved without a strike? What is the probability that a dispute which was resolved without a strike was related to wages? **[Eco. (H) I Sem. 2014]**

Q36. A firm produces steel pipes in plants with daily production volumes of 500, 1000 and 2000 units respectively. According to the past experience, it is known that the fraction of defective output produced by the 3 plants are respectively 5% (0.05), 8% (0.08) and 10% (0.10). If a pipe is selected from a day's total production and found to be defective, what is the probability that it came from the first plant? **[Eco. (H) 2000]**

Q37. In a railway reservation office, two clerks are engaged in checking reservation forms. On an average, the first checks 55% of the forms, while the second does the remaining. The first clerk has an error rate of 0.03 and second has an error rate of 0.02. A reservation form is selected at random from the total number of forms checked during a day, and is found to have an error. Find the probability that it was checked (i) by the first, (ii) by the second clerk. **[Eco. (H) 1999]**

Q38. By examining the chest X-ray, the probability that T.B. is detected when a person is actually suffering is 0.99. The probability that the doctor diagnoses incorrectly that a person has TB on the basis of X-ray is 0.001. In a certain city, 1 in 1000 persons suffers TB. A person selected at random and diagnosed to have T.B. what is the chance that he actual has TB.

Q39. The chances that a doctor will diagnose a disease correct is 60%. The chances that a patient will die by his treatment after correct diagnosis is 40% and the chances of death by wrong diagnosis is 70%. A patient of the doctor who had the disease was diagnosed correctly? **[Eco. (H) 2004]**

Q40. A test for detection of a particular disease is not foolproof. The test will correctly detect the disease 90% of the time, but will incorrectly detect the disease 1% of the time. For a large population of which an estimated 0.2% have the disease, a person is selected at random, given the test, and told that he has the disease. What are the chances that the person actually does have the disease ?

Q41. Let d_1, d_2, d_3 be three mutually exclusive diseases. Let S be the set of observable symptoms of these diseases. A doctor has the following information from a random sample of 5000 patients : 1800 had disease d_1, 2100 had disease d_2 and others had disease d_3. 1500 patients with disease d_1, 1200 patients with disease d_2 and 900 patients with disease d_3 showed the symptoms S. which of these diseases is the patient most likely to have ?

Q42. You ask your neighbor to water a sick plant while you are on a vacation. Without water the plant will die with a probability of 0.8, with water it will die with a probability of 0.16. You are 90% sure that your neighbor will remember to water the plant. **[Eco. (H) I Sem. 2014]**

(i) What is the probability that the plant will be alive when you return?

(ii) If the plant is dead upon your return, what is the probability that your neighbor forgot to water it?

Q43. In a test, an examinee either guesses or copies or knows the answer to a multiple-choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct, given that he copied it is 1/8. Find the probability that he knew the answer to question, given that he correctly answered it.

Q44. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Q45. A is known to speak truth 3 out of 5 times. He throws a die & reports that it is 1. Find the probability that it is actually 1.

Q46. A speaks the truth 2 out of 3 times and B 4 out of 5 times. They agree that from a bag 6 ball of different colours, a black ball has been drawn. Find the probability that the statement is true.

Q47. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both spades. Find the probability of the lost card being a spade.

Q48. You note that your officer is happy on 60% of your calls, so that you assign a probability of his being happy on your visit as 0.6 or 6/10. You have noticed that if he is happy, he accedes to your request with a probability of 0.4 or 4/10 whereas if he is not happy, he accedes to your request with a probability of 0.1 or 1/10 . you call one day and he accedes to your request. What is the probability that he is happy?

Q49. In a class of 75 students, 15 were considered to be very intelligent, 45 as medium and the rest below average. The probability that a very intelligent student fails in a viva voce examination is 0.005; the medium student failing has a probability 0.05; and the corresponding probability for a below average student is 0.15. If a student is known to have passed the viva-voce examination, what is the probability that he is below average? **[Eco. (H) 1997]**

Q50. A company launches an advertising campaign of its new product on TV, radio and in print media in an area where 30% watch TV, 50% listen to radio and the rest rely on the newspapers for all information. It is estimated that a person who sees the advertisement on TV will buy the product with probability 0.6. A person who has heard it on radio is expected to buy the product with probability 0.3 and seeing the advertisement in print media will convince a person to buy the product with probability 0.1. A consumer, chosen at random is found to have purchased the product, what is the probability she heard about the product on radio.**[Eco (H) 2007]**

Q51. If a machine is correctly set-up it produces 90% acceptable items. If it is incorrectly set-up it produces 40% acceptable items. Experience shows that 80% of the set-ups are correctly done.

(i) If after a certain set-up, out of the first two items produced, first is found to be acceptable and second unacceptable, what is the probability that the machine is correctly set-up?

(ii) If the machine produced first two items as acceptable, what is the probability that the machine is correctly set-up?

Q52. A company uses three different assembly lines – A_1, A_2 and A_3 to manufacture a particular component. Of those manufactured by line A_1, there are 5% that need reworking to remedy a defect, whereas 8% of A_2's components need reworking and 10% of A_3's need reworking. Suppose that 50% of all components are produced by line A_1, while 30% are produced by line A_2 and 20% come from line A_3. What is the probability that a randomly selected component :

(i) Came from line A_1 and needed rework?

(ii) Needed Reworking?

(iii) Came from line A_2, given that it needed reworking?

(iv) Came from line A_3, and needed reworking? **[Eco. (H) 2011]**

Q53. Three identical boxes are given. Box A contains 5 red and 3 white balls. Box B contains 3 red and 5 white balls. Box C contains 4 red and 4 white balls. A fair coin is tossed. If a 5 or 6 appears then a ball is randomly chosen from Box A. If a 4 appears then a ball is randomly chosen from Box B. Otherwise a ball is randomly selected from Box C.

(i) Find the probability that a red ball is chosen.

(ii) If a red ball is selected, what is the probability it came from Box B ?

[Eco. (H) I Sem. 2011]

Q54. Mina is hoping to get a summer job with a consulting firm. If her interview goes well she has a 70% chance of getting an offer, otherwise her chances of getting the

job drop to 20%. Unfortunately, Mina tends to stammer under stress so the likelihood of the interview going well is only 0.10.

(i) What is the probability that Mina gets the job?

(ii) If she gets the job what is the probability that her interview went well?

[Eco. (H) I Sem. 2012]

MA Entrance

Q1. English and American spellings are "rigour" and "rigor", respectively. A guest at a hotel writes this word and a letter taken at random from her spelling is found to be a vowel. If 40% of the guests are English and 60% are Americans, what is the probability that the writer is English? **[DSE MA Ent. Eco. 2001]**

[**Ans. :** 5/11]

Q2. Consider an examinee answering a multiple-choice examination. For a particular question with 4 choices, the probability that the examinee knows the answer is 1/3, the probability that she makes the correct choice given that she knows the answer is 1, and the probability that she makes the correct choice given that she does not know the answer is ¼. The probability that she knew the answer given that she has made the correct choice is **[DSE MA Ent. Eco. 2004]**

(a) 1/3 (b) 3/4 (c) 2/3 (d) 5/6

[**Ans. :** (c)]

Q3. There are three identical boxes, each with two drawers. Box A contains a gold coin in each drawer. Box B contains a silver coin in each drawer. Box C contains a gold coin in one drawer and a silver coin in another drawer. A box is chosen at random, a drawer opened and a gold coin is found. What is the probability that the chosen box is A? **[DSE MA Ent. Eco. 2004, 2005]**

(a) 2/3 (b) 1/3 (c) 1/2 (d) 3/4

[**Ans. :** (a)]

Q4. A blood test to detect the presence of a certain disease detects the disease in 99 out of every 100 patients who actually have the disease. But it falsely indicates the presence of the disease in 1 out of every 100 healthy persons. 1 percent of the population has the disease. The test is administered to a randomly selected person. What is the probability that she has the disease given that her result is positive?

(a) 2/3 (b) ½ (c) ¾ (d) 3/5

[**Ans. :** (b)] **[DSE MA Ent. Eco. 2005]**

Q5. A blood test detects a given disease with probability 8/10 given that the tested person actually has the disease. With probability 2/10, the test incorrectly shows the presence of the disease in a disease free person. Suppose 1/10 of the population has the disease. What is the probability that the person tested actually has the disease if the test indicates the presence of the disease?**[DSE MA Ent. Eco. 2006]**

(a) 1 (b) 9/13 (c) 4/13 (d) 7/13

[**Ans. :** (c)]

Q6. Suppose that 80% of all statisticians are shy, whereas only 15% of all economists are shy. Suppose also that 90% of the people at a large gathering are economists

and the other 10% are statisticians. If you meet a shy person at random at the gathering, what is the probability that the person is a statistician?

(a) 8/9 (b) 0.8 (c) 0.08 (d) 80/215

[**Ans. :** (d)] **[DSE MA Ent. Eco. 2008]**

Answers of Exercise 7

1.	½,	**2.**	19/42,	**3.**	1/2,	**5.**	193/792,	**6.(i)**	1/250,
(ii)	8,	**7.**	15/29,	**8.**	25/52,	**9.**	61/140 & 40/61,		
10.	36/61,	**11.**	48/133,	**12.**	33/118,	**13.**	1/40,	**14.**	27/83,
15.	27/83,	**16.**	3/7,	**17.(i)**	0.57,	**(ii)**	0.33,	**(iii)**	0.1,
18.	8/11,	**19.**	¾,	**20.**	0.41, 0.6376,	**21.**	41/69,	**22.**	1/5,
23.	1/10,	**24.**	5/61,	**25.**	2/23,	**27.**	0.44 approx.,		
28.	½,	**29.**	2/9,	**30.**	3/7,	**31.**	2/3,	**32.**	1/52,
33.(i)	3/19,	**(ii)**	6/19,	**(iii)**	10/19,	**34.**	1/13,	**36.**	5/61,
37(i).	0.643,	**(ii)**	0.3529,	**38.**	110/221,	**39.**	0.46,	**40.**	0.15,
41.	d_1,	**43.**	24/29,	**44.**	3/8,	**45.**	3/13,	**46.**	1/10,
47.	11/50,	**48.**	6/7,	**49.**	0.18,	**50.**	3/7,	**51.(i)**	0.9,
(ii)	0.95,	**52.(i)**	5/100,	**(ii)**	6.9/100,	**(iii)**	24/69,	**(iv)**	20/69
53.(i)	25/48,	**(ii)**	3/25						

Basic Concepts

1. **Mathematical Expectation :** Let X be a random variable which takes $x_1, x_2, x_3, \ldots, x_n$ mutually exclusive values with respective probabilities $p_1, p_2, p_3, \ldots, p_n$. Then the expected value or mean of X is given as under :

$E(X) = p_1 . x_1 + p_2 . x_2 + p_3 . x_3 + \ldots + p_n . x_n$

Exercise 8

Q1. An investor rents a stall for the summer and has to choose between two projects, A and B. He can either run an ice-cream parlour (A) or a hot soup and snacks bar (B). if the summer is very hot he can make an estimated profit of Rs. 13,000 from A or Rs. 2,000 from B. If the summer is mild then the estimated profits from A and B are Rs. 2,000 and Rs. 10,000 respectively. If the summer is hot then the respective estimated profits for A and B are Rs. 7,000 and Rs. 6,000. If the probability of a very hot summer is 30% and that for a mild summer is 40% which project should he choose, if his aim is to maximize expected profits. **[Eco. (H) I Sem. 2011]**

Q2. A charitable organization raises funds by selling 2,000 raffle tickets for a 1st prize worth Rs. 500 and a 2nd prize worth Rs. 100. What is the mathematical expectation of a person who buys a ticket. **[Eco. (H) I Sem. 2012]**

Q3. From a bag containing 4 white and 6 red balls, three balls are drawn at random :

(i) Find the expected number of white balls drawn

(ii) If each white ball drawn carried a reward of Rs. 4 and each red ball Rs.6, find the expected reward of the draw. **[B. Com. (H) 2002]**

Q4. A box contains 8 tickets, 3 of them carry a prize of Rs. 5 each and the remaining 5 a prize of Rs. 2 each. **[B. Com. (H) 2005(C)]**

(i) If one ticket is drawn at random, what is the expected value of the prize?

(ii) If two tickets are drawn at random, what is the expected value of the prize?

Q5. A and B throw one die for a prize of Rs. 11, which is to be won by the player who first throws 6. If A has the first throw, what are their respective expectations? **[Eco (H) 1994]**

Q6. A bag contains 2 white balls and 3 black balls. Four persons A, B, C, D in the order named each draws one ball and does not replace it. The first to draw a white ball receives Rs. 20. Determine their expectations? **[Eco (H) 1995]**

Q7. The probability known is 0.99 that a 30 years old man will survive one year more. An insurance company offers to sell such a man a Rs. 10,000 one-year term life insurance policy at premium of Rs 110. What is the insurance company's expected gain? **[B. Com. (H) 2005(C)]**

Q8. A contractor spends Rs. 3,000 to prepare for a bid on a construction project which, after deducting manufacturing expenses and the cost of bidding, will yield a profit of Rs. 25,000 if the bid is won. If the chance of winning the bid is ten percent, compute his expected profit and the state the likely decision on whether to bid or not to bid. **[Eco (H) 1998]**

Q9. A contractor spends Rs. 3,000 to prepare for a bid on a construction project which after deducting manufacturing expenses and the cost of bidding, will yield a profit of Rs. 25,000 if the bid is won. The chance of winning the bid is only 10%.

(i) Compute his expected profit and state the likely decision on whether to bid or not to bid.

(ii) If the contractor wishes his expected profit to be Rs. 300, what amount of adjustment he would demand in the cost of bidding? **[Eco. (H) II Sem. 2014]**

Q10. If a man purchases a raffle ticket, he can win a first prize of Rs. 5,000 or a second prize of Rs. 2,000 with probabilities 0.001 and 0.003. What should be a fair price to pay for the ticket? **[Eco (H) 2001]**

Q11. A and B plays for a prize. A is to throw a die first and win if he throws a 6. if he fails, B is to throw and win if he throws 6 or 5. If B fails, A is to throw again and win if he gets 6, 5 or 4. The game continues in this manner till it is won. The winner is to get a cash prize of Rs. 3240. What are the respective expected winning. **[Eco (H) 2006]**

Q12. A maker of soft drinks is considering the introduction of a new brand. He expects to sell 50,000 to 1,00,000 bottles of its soft drink in a given period according to the following probability distribution : **[B. Com. (H) 2005(R)]**

No. of Bottles Sold (in '000)	:	50	60	70	80	90	100
Probability	:	0.13	0.20	0.35	0.20	0.08	0.02

If the product is launched he will have to incur a fixed cost of Rs. 48,000. However, each bottle sold would give him a profit of Rs. 1.25. Should he introduce the new brand?

MA Entrance

Q1. The average travel time to a distant city is w hours by train or z hours by bus. A man cannot decide whether to take the train or the bus, so he chooses a coin. What is his expected travel time? **[DSE MA Ent. Eco. 2007]**

(a) $2(w + z)$ (b) $2(w + z) / (w – z)$

(c) $(w + z + 2) / 2$ (d) None of the above

[**Ans. :** (d)]

Q2. Let X denote the absolute value of the difference between the numbers obtained when two dice are tossed. The expectation of X is : **[DSE MA Ent. Eco. 2009]**

(a) $1\frac{32}{36}$ (b) $1\frac{33}{36}$ (c) $1\frac{17}{18}$ (d) $1\frac{16}{18}$

[**Ans. :** (c)]

Q3. Suppose that there is only one future period and the (presently unknown) state of the world in that period can be either s_1 or s_2. The future return on a share of a given company is 5 in state s_1 and -1 in state s_2. The future return on a government bond is 1 independent of the state . Suppose a third asset is offered on the market whose return is 3 in state s_1 and 0 in state s_2. The current price of the stock and the bond are 3 and 1 respectively. If the price of the new asset rules out the possibility of arbitrage profit (which arises when portfolio of assets that are identical in terms of returns have different prices), what is the price of the new asset? **[DSE MA Ent. Eco. 2010]**

(a) It depend on the probabilities of future states (b) Strictly between 2 and 3

(c) Strictly between 1 and 2 (d) 2

[**Ans. :** (d)]

Answers of Exercise 8

1. Project A, **2.** 0.3, **3.(i)** 2/5, **(ii)** 78/5, **4.(i)** 25/8, **(ii)** 35/9,
5. 6 & 5, **6.** Rs. 8, Rs. 6, Rs. 4 & Rs. 2, **7.** Rs. 10,
8. – Rs. 200, not to bid, **10.** Rs. 11, **11.** Rs. 521.6 & Rs. 478.4,
12. Yes