Chapter 3

Utility

Learning Objectives :

After learning this chapter you will understand :

- **Meaning of Utility.**
- **Utility Function.**
- **Ordinal Utility.**
- **Monotonic Transformations.**
- **Cardinal Utility.**
- **Some Examples of Utility Functions.**
- **Marginal Utility.**
- **Marginal Rate of Substitution.**

Basic Concepts

1. Meaning : Utility means ability of a good or service to satisfy one or more needs or wants of a consumer, in other words, utility means satisfaction which a consumer derive from commodities and services by purchasing different units of money.

2. Utility Function : While it is natural to think about preferences, it is often more convenient to associate different numbers to different goods, and have the agent choose the good with the highest number. These numbers are called utilities. In turn, a utility function tells us the utility associated with each good $x \in X$, and is denoted by $u(x) \in \mathfrak{R}$. We say a utility function $u(x)$ represents an agent's preferences. That is, a bundle (x_1, x_2) is preferred to a bundle (y_1, y_2) if and only if the utility of (x_1, x_2) is larger than the utility of (y_1, y_2).
Symbolically, $(x_1, x_2) \succ (y_1, y_2)$ if and only if $u(x_1, x_2) > u(y_1, y_2)$.
This means than an agent makes the same choices whether she uses her preference relation, $\succ$, or her utility function $u(x)$.

> *Note* : We can find a numerical value u that represents the preferences of our consumer if and only if the preferences are complete and transitive.

Utility Representation Theorem : Suppose the agent's preferences, $\geq$, are complete and transitive, and that X is finite. Then there exists a utility function $u(x) : X \to \mathfrak{R}$ which represents $\geq$.
This theorem says that if an agent has complete and transitive preferences then we can associate these preferences with a utility function. Intuitively, the two axioms allow us to rank the goods under consideration. For example, if there are 10 goods, then we can say the best has a utility $u(x) = 9$, the second best has $u(x) = 8$, the third best has $u(x) = 7$ and so on.

3. Ordinal Utility : The only property of a utility assignment that is important is how it orders the bundles of goods. The magnitude of the utility function is only important insofar as it ranks the different consumption bundles; the size of the utility difference between any two consumption bundles doesn't matter. Because of this emphasis on ordering bundles of goods, this kind of utility is referred to as ***ordinal utility***.

Since only the ranking of the bundles matters, there can be no unique way to assign utilities to bundles of goods. If we can find one way to assign utility numbers to bundles of goods, we can find an infinite number of ways to do it.

4. Monotonic Transformation : A monotonic transformation is a way of transforming one set of numbers into another set of numbers in a way that preserves the order of the numbers. We typically represent a monotonic transformation by a function $f(u)$ that transforms each number u into some other number $f(u)$, in a way that preserves the order of the numbers in the sense that $u_1 > u_2$ implies $f(u_1) > f(u_2)$. A monotonic transformation and a monotonic function are essentially the same thing.

The rate of change of $f(u)$ as u changes can be measured by looking at the change in f between two values of u, divided by the change in u :

$$\frac{\Delta f}{\Delta u} = \frac{f(u_2) - f(u_1)}{u_2 - u_1}$$

For a monotonic transformation, $f(u_2) - f(u_1)$ always has the same sign as $u_2 - u_1$. Thus a monotonic function always has a positive rate of change. This means that the graph of a monotonic function will always have a positive slope.

Examples of monotonic transformations are multiplication by a positive number (e.g., $f(u) = 3u$), adding any number (e.g., $f(u) = u + 17$), raising u to an odd power (e.g., $f(u) = u^3$), and so on.

A ***monotonic transformation of a utility function*** is a utility function that represents the same preferences as the original utility function, *i.e.*, If $f(u)$ is any monotonic transformation of a utility function that represents some particular preferences, then $f(u(x_1, x_2))$ is also a utility function that represents those same preferences.

> *Note* : What we are calling a "monotonic transformation" is, strictly speaking, called a "positive monotonic transformation," in order to distinguish it from a "negative monotonic transformation," which is one that reverses the order of the numbers. If we raise u to an even power, *e.g.*, $f(u) = u^2$, then it is a monotonic transformation only for positive values of u and not for negative values.

5. **Cardinal Utility :** Cardinal utility means satisfaction that can be measured in numbers such as 1, 2 and 3 etc. In a theory of cardinal utility, the size of the utility difference between two bundles of goods is supposed to have some sort of significance. While ordinal utility refers to satisfaction which can't be measured in numbers. The concept of cardinal utility was used by Marshal to define Consumer's Equilibrium. Cardinal utility means consumer could measure the satisfaction derived by the consumption of any goods or services in terms of numbers and unit of that measurement is Utils or the Money. Whereas ordinal utility means giving rank to the utility derived by the consumption of goods and services. The concept or ordinal utility is more realistic and better than cardinal utility.

6. **Some Examples of Utility Functions :** If you are given a utility function, $u(x_1, x_2)$, it is relatively easy to draw the indifference curves: you just plot all the points (x_1, x_2) such that $u(x_1, x_2)$ equals a constant, say k. In mathematics, the set of all

(x_1, x_2) such that $u(x_1, x_2)$ equals a constant is called a level set. For each different value of the constant, you get a different indifference curve.

(i) **Symmetric Cobb Douglas :** Suppose $u(x_1, x_2) = x_1x_2$. We calculate the marginal rate of substitution two ways. First, we can use equation MRS = $-\frac{dx_2}{dx_1}$ to derive MRS. We know that a typical indifference curve is just the set of all x_1 and x_2 such that $k = x_1x_2$ for some constant k. Solving for x_2 as a function of x_1, we see that a typical indifference curve has the formula :

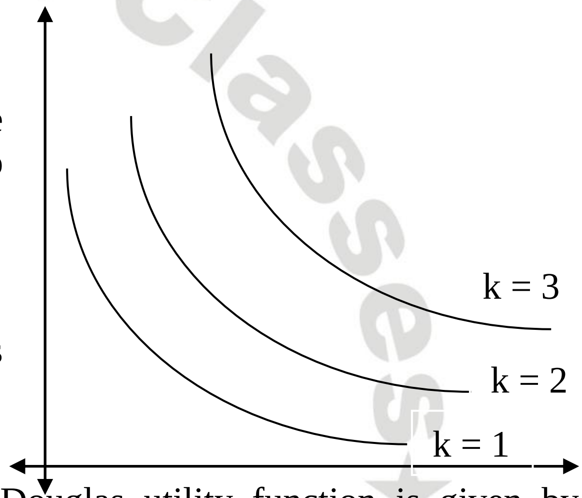
$$x_2 = \frac{k}{x_1}$$

Now, MRS = $-\frac{dx_2}{dx_1} = \frac{k}{x_1^2}$

We can now verify that preferences are convex. Differentiating again with respect to x_1, we get

$$\frac{d}{dx_1}(MRS) = -\frac{2k}{x_1^3}$$

Which is negative as required. This curve is depicted as under for k = 1, 2, 3 · · ·



(ii) **Cobb Douglas Preferences :** The Cobb Douglas utility function is given by $U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$ for α > 0, β > 0.

The Cobb Douglas indifference curve has equation $x_1^{\alpha} x_2^{\beta} = k$. Rearranging, $x_2 = k^{1/\beta} x_1^{-\alpha/\beta}$. These indifference curves look like those in the above figure. The marginal utilities are :

$$MU_1 = \alpha x_1^{\alpha-1} x_2^{\beta} \quad \text{and} \quad MU_2 = \beta x_1^{\alpha} x_2^{\beta-1}$$

As a result the MRS is,

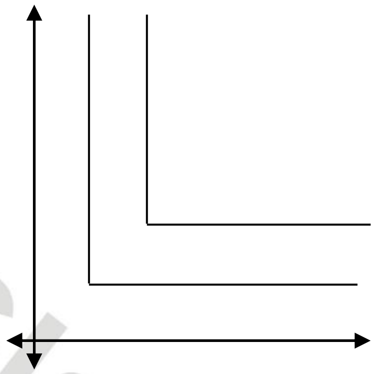
$$MRS = \frac{MU_1}{MU_2} = \frac{\alpha x_2}{\beta x_1}$$

If for the Cobb-Douglas utility function $U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$, we have α > β then the indifference curves will be steeper, whereas if we have α < β, then the indifference curves will be flatter. The indifference curves represented by Cobb-Douglas Preferences are nice convex indifference curves and they are well behaved.

The monotonic transformation of Cobb-Douglas utility function will represent exactly the same preferences and we can always take a monotonic transformation of the Cobb-Douglas utility function that make the sum of the exponents equal to 1.

(iii) **Perfect Complements :** Suppose an agent always consumes a hamburger patty with two slices of bread. If she has 5 patties and 15 slices of bread, then the last 5 slices are worthless. Similarly, if she has 7 patties and 10 slices of bread, then the last 2 patties are worthless. In this case, the agent's preferences can be represented by the utility function :

$$U(x_1, x_2) = \min\{2x_1, x_2\}$$



where x_1 are patties and x_2 are slices of bread. Note the 2 goes in front of the number of patties because, intuitively speaking, each patty is twice as valuable as a piece of bread.

In general, preferences are perfect complements when they can be represented by a utility function of the form

$$U(x_1, x_2) = \min\{\alpha x_1, \beta x_2\}$$

The resulting indifference curves are L shaped, as shown in figure, with the kink along the line $\alpha x_1 = \beta x_2$. Note that the indifference curve is not strictly decreasing along the bottom of the L. This is because these preferences do not quite obey the monotonicity condition, when the agent has 7 patties and 10 slices of bread, an extra patty does not strictly increase her utility.

The MRS in this example is a little odd. When $\alpha x_1 > \beta x_2$,

$$MRS = \frac{MU_1}{MU_2} = \frac{0}{\beta} = 0$$

When $\alpha x_1 > \beta x_2$,

$$MRS = \frac{MU_1}{MU_2} = \frac{\alpha}{0} = \infty$$

At the kink, when $\alpha x_1 = \beta x_2$, then MRS is not defined because the indifference curve is not differentiable.

(iv) **Perfect Substitutes :** Suppose an agent is buying food for a party. She wants enough food for her guests and considers 3 hamburgers to be equivalent to one pizza. Since each pizza is three times as valuable as a hamburger, her preferences can be represented by the utility function :

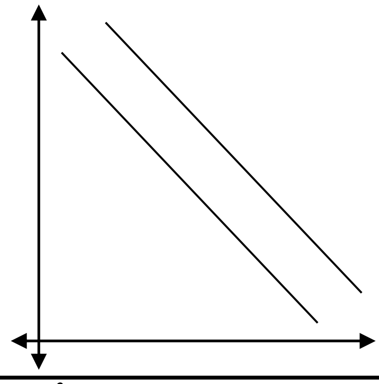
$$U(x_1, x_2) = x_1 + 3x_2$$

where x_1 are hamburgers and x_2 are pizzas.

In general, preferences are perfect substitutes when they can be represented by a utility function of the form :

$$U(x_1, x_2) = \alpha x_1 + \beta x_2$$

The resulting indifference curves are straight lines, as shown in given figure. As a result, preferences are only weakly convex. The marginal rate of substitution is :



$$MRS = \frac{MU_1}{MU_2} = \frac{\alpha}{\beta}$$

That is, the MRS is independent of the number of goods consumed.

(v) **Constant Elasticity of Substitution (CES) Preferences :** CES preferences have the form :

$$U(x_1, x_2) = \frac{x_1^{\delta}}{\delta} + \frac{x_2^{\delta}}{\delta}$$

where $\delta \neq 0$ and $\delta < 1$.

This utility function can approximate the above examples. As $\delta \to 0$ the limit of the above utility function becomes

$$U(x_1, x_2) = \ln x_1 + \ln x_2$$

which is the same as Cobb-Douglas with equal exponents. As $\delta \to 1$, the preferences approximate perfect substitutes. As $\delta \to -\infty$, the preferences approximate perfect complements. The MRS is :

$$MRS = \frac{MU_1}{MU_2} = \frac{\delta x_1^{\delta-1}}{\delta x_2^{\delta-1}} = \frac{x_2^{1-\delta}}{x_1^{1-\delta}}$$

The last expression is convenient since $1 - \delta > 0$. Substituting for x_2 in this equation and differentiating, one can show that MRS is decreasing in x_1, so the preferences are convex.

(vi) **Quasi-linear Preferences :** An agent has quasi-linear preferences if they can be represented by a utility function of the form :

$$U(x_1, x_2) = v(x_1) + x_2$$

Quasi-linear preferences are linear in x_2, so the marginal utility is constant. These preferences are often used to analyze goods which constitute a small part of an agent's income; good x_2 can then be thought of as "general consumption".

The marginal rate of substitution equals

$$MRS = \frac{MU_1}{MU_2} = \frac{v'(x_1)}{1} = v'(x_1)$$

Observe that MRS only depends on x_1, and not x_2. This means that the indifference curves are vertical parallel shifts of each other, as shown in given figure. As a consequence, preferences are convex if and only if $v(x_1)$ is a concave function, so the marginal utility of x_1 decreases in x_1.

As we will see later, quasilinear preferences have the attractive property that the consumption of x_1 is independent of the agent's income (ignoring boundary constraints). This makes the consumer's problem simple to analyze and provides an easy way to calculate consumer surplus.

(vii) **Additive Preferences :** Additive preferences are represented by a utility function of the form :

$$U(x_1, x_2) = v_1(x_1) + v_2(x_2)$$

The key property of additive preferences is that the marginal utility of x_i only depends on the amount of x_i consumed. As a result, the marginal rate of substitution is

$$MRS = \frac{MU_1}{MU_2} = \frac{v_1^{/}(x_1)}{v_2^{/}(x_2)}$$

For example, suppose we have

$$U(x_1, x_2) = x_1^2 + x_2^2$$

Differentiating, $MU_i = 2x_i$, so the marginal utility of each good is increasing in the amount of the good consumed. For example, one could imagine the agent becomes addicted to either good. As shown in given figure, these preferences are concave. One can see this formally by showing the MRS is increasing in x_1 along an indifference curve. Differentiating,

$$MRS = \frac{MU_1}{MU_2} = \frac{2x_1}{2x_2} = \frac{x_1}{x_2}$$

The equation of an indifference curve is $x_1^2 + x_2^2 = k$. Rearranging, $x_2 = \left(k - x_1^2\right)^{1/2}$. Substituting this value into MRS above, we get :

$$MRS = \frac{x_1}{\left(k - x_1^2\right)^{1/2}}$$

Which is increasing in x_1.

7. **Marginal Utility :** The rate of change in consumer's total utility when he is allowed to consume more of good 1 is called marginal utility with respect to good1, keeping good 2 as constant. We write it as MU_1 and it is considered as the following ratio :

$$MU_1 = \frac{\Delta U}{\Delta x_1}$$

This ratio measures the rate of change in utility (ΔU) associated with a small change in the quantity of good 1 keeping good 2 as constant.

To calculate the change in the utility associated with a small change in the consumption of good1, we can just multiply the change in consumption by the marginal utility of the good as $\Delta U = MU_1.\Delta x_1$. Similarly, the marginal utility with respect to good 2 can be defined as :

$$MU_2 = \frac{\Delta U}{\Delta x_2}$$

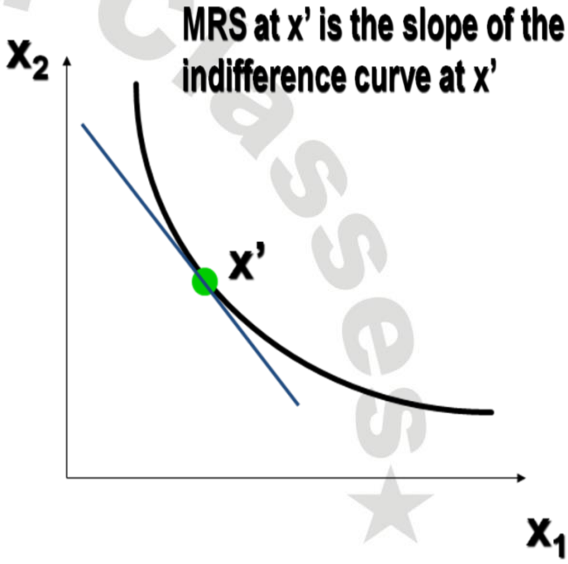
To calculate the change in the utility associated with a small change in the consumption of good2, we can just multiply the change in consumption by the marginal utility of the good as $\Delta U = MU_2.\Delta x_2$.

Now, if the utility function is $U = U(x_1, x_2, ..., x_k)$ then the change in total utility can be found as :

$$\Delta U = MU_1.\Delta x_1 + MU_2.\Delta x_2 + ...+ MU_k.\Delta x_k$$

$$\Rightarrow \quad \Delta U = \frac{\Delta U}{\Delta x_1}.\Delta x_1 + \frac{\Delta U}{\Delta x_2}.\Delta x_2 + ...+ \frac{\Delta U}{\Delta x_2}.\Delta x_k$$

8. **Marginal Utility and MRS :** The slope of an indifference curve is its marginal rate-of-substitution (MRS). Concept of MRS changes level of only two commodities (say, x_1 and x_2), keeping household indifferent (ΔU = 0). The MRS can be portrayed as the slope of the indifference curve and since on an indifference curve we have ΔU = 0. Therefore,



$$\Delta U = MU_1.\Delta x_1 + MU_2.\Delta x_2 = 0$$

So, $MRS = \dfrac{\Delta x_2}{\Delta x_1} = -\dfrac{MU_1}{MU_2}$.

The algebraic sign of the MRS is negative, which mean if you get more of good 1 you have to get less of good 2 in order to keep the same level of utility. However in calculations generally we take absolute value of MRS, for the sake of simplicity.

Note **Monotonic transformation of a utility function does not change its MRS.**

Theory Questions

Q1. What do you mean by monotonic transformation? Which of the following are monotonic transformations :

(a) $u = v^2$ for $v > 0$, (b) $u = v^2$ for $v < 0$, (c) $u = v^2$,
(d) $u = 3v - 4$, (e) $u = 1/v^2$, (f) $u = -1/v^2$, (g) $u = \ln(v)$,
(h) $u = e^v$, (i) $u = -e^{-v}$,

Ans. **(a)** Yes, **(b)** No, **(c)** No, **(d)** Yes, **(e)** No, **(f)** No, **(g)** Yes, **(h)** Yes, **(i)** Yes.

Q2. Given $U = \min(x_1, x_2)$, what kind of preferences are represented by U, draw some indifference curves represented by these preferences.

Q3. What kind of preferences are represented by the following utility functions, where x_1, x_2 denote the amount of good 1 and good 2 respectively :

(i) $U(x_1, x_2) = x_1 + x_2$, (ii) $U(x_1, x_2) = (x_1 + x_2)^2$

(iii) $U(x_1, x_2) = x_1 + \ln x_2$

Ans. **(i)** Perfect Substitutes, **(ii)** Perfect Substitutes,

(iii) Quasi Linear.

Q4. What kind of preferences are represented by a utility function of the form $U(x_1, x_2) = \sqrt{x_1 + x_2}$? What about the utility function $V(x_1, x_2) = 13x_1 + 13x_2$?

Ans. Perfect Substitutes.

Q5. What kind of preferences are represented by a utility function of the form $U(x_1, x_2) = x_1 + \sqrt{x_2}$? Is the utility function $V(x_1, x_2) = x_1^2 + 2x_1\sqrt{x_2} + x_2$ a monotonic transformation of $U(x_1, x_2)$?

Ans. Quasi-linear Preferences, Yes.

Q6. What kind of preferences are represented by a utility function of the form $U(x_1, x_2) = \sqrt{x_1 x_2}$? Is the utility function $V(x_1, x_2) = x_1^2.x_2$ a monotonic transformation of $U(x_1, x_2)$? Is the utility function $W(x_1, x_2) = x_1^2.x_2^2$ a monotonic transformation of $U(x_1, x_2)$?

Ans. Cobb-Douglas Preferences, No, Yes.

Q7. What kind of preferences are represented by the utility function $U(x_1, x_2) = x_1^{1/3} x_2^{2/3}$. Are the following monotonic transformation of the above stated utility function :

(i) $U(x_1, x_2) = x_1^{2/3} x_2^{2/3}$, (ii) $U(x_1, x_2) = x_1^{2/3} x_2^{1/3}$

(iii) $U(x_1, x_2) = x_1^{2/3} x_2^{4/3}$

Q8. In a two commodity world (x_1, x_2), specify utility functions where

(a) x_1 and x_2 are perfect substitutes with one unit of x_1 equivalent to three units of x_2.

(b) x_1 and x_2 are perfect complements and one unit of x_1 is always used with four units of x_2. **[Eco. (H) III Sem. 2013]**

Q9. A consumer spends all his money on coffee and sugar. He only drinks his coffee with two spoons of sugar and only consumes sugar if he drinks coffee.

(a) Write his utility function.

(b) Graph his indifference curves. **[Eco. (H) III Sem. 2013]**

Q10. Show that a Cobb-Douglas consumer spends a fixed proportion of his income on each good in his consumption bundle.

Q11. Explain the implications of Quasi linear preferences?

Q12. Consider the following utility functions :

(i) $U(x, y) = x.y$, (ii) $U(x, y) = x^2.y^2$, (iii) $U(x, y) = \ln x + \ln y$

Show that each of these has a diminishing MRS but that they exhibit constant, increasing and decreasing marginal utility, respectively. What do you conclude?

[**Hint :** The shape of marginal utility function is not necessarily an indicator of convexity of indifference curves]

Q13. Why taking a monotonic transformation of a utility function doesn't change the marginal rate of substitution?

Q14. Depict for the following utility function that by taking its monotonic transformation the MRS does not change : $U = x^{1/3}y^{2/3}$.

Q15. Briefly explain the assumptions of *monotonicity* and *convexity* of consumer preferences and their implications for the shapes of the associated indifference curves. **[Eco. (H) III Sem. 2014]**

Numerical Problems

Q1. A consumer is willing to trade 3 units of X for 1 unit of Y when she has 6 units of X and 5 units of Y. She is also willing to trade 6 units of X for 2 units of Y when she has 12 units of X and 3 units of Y. She is indifferent between bundle (6, 5) and (12, 3). What is the utility function for goods X and Y?
[**Ans. :** $U(x, y) = \alpha x + 3\alpha y$, MRS = -1/3]

Q2. A consumer is willing to trade 4 units of X for 1 unit of Y when she is consuming bundle (8, 1). She is also willing to trade in 1 unit of X for 2 units of Y when she is consuming bundle (4, 4). She is indifferent between these two bundles. Assuming that the utility function is Cobb-Douglas of the form $U(x, y) = x^{\alpha} y^{\beta}$, where α and β are positive constants. What is the utility function for this consumer?
[**Ans. :** $U(x, y) = x^{2/3}y^{1/3}$]

Q3. Nikhil consumes two goods x_1 and x_2. He has the utility function $u(x_1, x_2) = x_1.x_2$. Determine the utility for the consumption bundle (40, 5). Also, draw some of his indifference curves. Mona offers to Nikhil to give 15 units of x_2 if Nikhil gives her 25 units of x_1, should Nikhil make this trade? **[WB 4.1]**

Q4. Use separate graphs to draw indifference curves for each of the following utility functions and determine whether they have convex indifference curves :

(i) $U(x, y) = \min(2x + y, 2y + x)$ (ii) $U(x, y) = \max(2x + y, 2y + x)$

(iii) $U(x, y) = 3x + y$ (iv) $U(x, y) = \sqrt{x.y}$

(v) $U(x, y) = \sqrt{x} + y$ (vi) $U(x, y) = \sqrt{x^2 - y^2}$

(vii) $U(x, y) = \min(2x - y, 2y - x)$ (viii) $U(x, y) = x^2 + y^2$

Ans. **(i)** Yes, **(ii)** No, **(iii)** No, **(iv)** Yes, **(v)** Yes, **(vi)** No, **(vii)** Yes, **(viii)** No.

Q5. Which of the following utility functions represent consumer preferences that satisfy the assumption of strict convexity? **[Eco. (H) III Sem. 2014]**

(a) $U(x, y) = xy$ (b) $U(x, y) = x^2 + y^2$

(c) $U(x, y) = \max(x, y)$

Draw the indifference map and explain.

Q6. Ram consumes two commodities X and Y and his utility function is Min($x + 2y$, $y + 2x$).

(a) If Ram has the consumption bundle (8, 2), draw his indifference curve which pass through the point (8, 2).

(b) Draw indifference curve along which the utility of Ram is 6?

(c) At the point where Ram has 5 units of X and 2 units of Y, how many units of X would he be willing to trade for one unit of Y?

Q7. Reeta has strong preference for having equal number of apples and oranges. Her utility function is of the form $u(x, y) = \min\{2x - y, 2y - x\}$, where x represents the number of apples and y represents the number of oranges.

(a) Draw indifference curve along which the utility of Reeta is 10.

(b) When $\min\{2x - y, 2y - x\} = 2y - x$, does she has more apples or more oranges?

(c) When $\min\{2x - y, 2y - x\} = 2x - y$, does she has more apples or more oranges?

(d) If Reeta has 10 apples and 9 oranges and she gets 5 more oranges, will her utility increase or decrease?

(e) If Reeta has 16 apples and the number of oranges is more than the number of apples. She thinks that her utility is exactly as good as having 10 apples and 10 oranges, then determine the number of oranges which she has?

(f) If Reeta has 16 apples and the number of oranges is less than the number of apples. She thinks that her utility is exactly as good as having 10 apples and 10 oranges, then determine the number of oranges which she has?

Q8. Use separate graphs to draw indifference curves for each of the following utility functions :

(i) $U(x, y) = \min(2x + y, 2y + x)$

(ii) $U(x, y) = \max(2x + y, 2y + x)$

Are these preferences convex? Why?

Q9. Ruchika's preferences over bundles that contain non-negative amounts of x_1 and x_2 are represented by the utility function $U(x_1, x_2) = x_1^2 + x_2^2$. Draw a few of her indifference curves. What kind of geometric figures are they? Does Ruchika have convex preferences?

Ans. Quarter circles centred at origin. No, she doesn't have convex preferences.

Q10. Picabo, an aggressive skier, spends her entire income on skis and bindings. (Binding are the mechanism by which skiers attach their boots to the skis.)

(i) If Picabo wears out one pair of bindings for every one pair of skis, graph her indifference curves for skis and bindings, illustrating bindings on the horizontal axis and skis on the vertical axis.

(ii) If Picabo wears out two pairs of bindings for every one pair of skis, graph her indifference curves for skis and bindings, illustrating bindings on the horizontal axis and skis on the vertical axis.

[**Hint :** Perfect Complements]

Q11. Paula, a former actress, spends all her income attending plays and movies. She likes plays exactly three times as much as she likes movies. Graph Paula's indifference curves, illustrating plays on the horizontal axis and movies on the vertical axis.

[**Hint :** Perfect Substitutes]

Q12. Mohit consumes soft drink which is available in ½ litre and 1 litre bottles. He is not concerned about the size of the bottle, he is concerned only about how much soft drink he has to consume.

(a) Write Mohit's utility function taking ½ litre bottle as x and 1 litre bottle as y.

(b) On the graph, draw some of his indifference curves.

(c) Would the utility function $u = 100x + 200y$ represent Mohit's preferences?

(d) Would the utility function $u = \{5x + 10y\}^2$ represent Mohit's preferences?

(e) Would the utility function $u = x + 3y$ represent Mohit's preferences?

Ans. **(a)** $U(x, y) = x + 2y$, **(c)** Yes, **(d)** Yes, **(e)** No.

[**Hint :** Perfect substitutes, slope -1/2]

Q13. Rohit consumes soft drink which is available in ½ litre and 1 litre bottles. He allows himself only half litre of soft drink at a time, so if he has a bottle of 1 litre he will throw ½ litre in the sink.

(a) Write Rohit's utility function taking ½ litre bottle as x and 1 litre bottle as y.

(b) On the graph, draw some of his indifference curves.

(c) Would the utility function $u = \{x + y\}^2$ represent Rohit's preferences?

Ans. **(a)** $U(x, y) = x + y$, **(c)** Yes.

[**Hint :** Perfect substitutes, slope -1]

Q14. A consumer who always consumes two teaspoons of sugar with each cup of tea, after heeding his doctor's advice now always consumes one teaspoon of sugar with each cup of tea. How does this change his utility function and his indifference curves between sugar and cups of tea? **[Eco. (H) III Sem. 2015]**

Q15. $U(x_1, x_2) = 4\sqrt{x_1} + x_2$. If $x_1 = 9$, $x_2 = 10$ find total utility. If initially 81 units of x_1 and 14 units of x_2 were being consumed, how much x_2 an individual is willing to give up to consume 40 more x_1.

Ans. : U = 22, $\Delta x_2 = -80/9$.

[**Hint :** Quasi-linear Preferences]

Q16. The utility function of Rohan is $u(x_1, x_2) = (x_1 + 2)(x_2 + 6)$.

(a) Determine the slope of his indifference curve at the point where his consumption bundle is (4, 6). Also, draw the IC which passes through the point (4, 6).

(b) Find at least one more point which lies on the indifference curve passing through the point (4, 6). Determine the equation of the indifference curve passing through the point (4, 6).

(c) Rohan has the bundle (4, 6) and Rosy offers him 9 units of x_2 in exchange of 3 units of x_1. Should Rohan make this trade?

(d) Rohan has the bundle (4, 6) and Rosy offers him 3 units of x_2 in exchange of 1 units of x_1. Should Rohan make this trade? If yes what will be his consumption bundle after this trade?

(e) Rohan has the bundle (4, 6) and Rosy offers him 6 units of x_2 in exchange of 2 units of x_1. Should Rohan make this trade? If yes what will be his consumption bundle after this trade?

Q17. Charu has the utility function of the form $u(x, y) = \max\{x, 2y\}$.

(a) Charu has the consumption bundle (10, 5). Draw the indifference curve passing through this point. Can you determine the MRS of such IC?

(b) Does Charu has convex preferences?

Q18. Consider the utility function U = x^2y^2. What is the marginal utility of good X? what is the marginal utility of good Y? Also find the marginal rate of substitution between goods X and Y?

Ans. $MU_x = 2xy^2$, $MUy = 2x^2y$, $MRS_{xy} = y/x$

Q19. Given $U = \min(x_1, x_2)$, obtain MU_{x_1} and depict it graphically as a function of x_1.

Q20. Consider the utility function $U(x, y) = 4x^2 + 6y$, and examine whether the assumption of 'more is better' is satisfied for each good?**[Eco. (H) III Sem. 2013]**

Q21. An individual's marginal utilities for commodity X and Y are given as :

$MU_x = 40 - 5x$, $MU_y = 20 - 3y$

What is the marginal rate of substitution if the consumer is consuming 3 units of X and 5 units of Y?

Ans. MRS = 5

Q22. A consumer's utility function is $\sqrt{x} + 2\sqrt{y}$ and money income is ` 10. Find MRS_{xy}.

Ans. $MRS = \frac{1}{2}\sqrt{\frac{y}{x}}$

Q23. Which of the following utility functions satisfies strict convexities?

(a) $U(x, y) = 3x^2 + (1/3)y^2$ (b) $U(x, y) = \min(2x + 3, y)$

Also, draw the indifference map for each of the above case.

[Eco. (H) III Sem. 2016]

Questions from Recent Eco (H) Examinations

Q1. What do assumptions of monotonicity and convexity imply about the shape of indifference curves? Does the utility function $u(x, y) = x + y^2$ satisfy these assumptions? **[Eco. (H) III Sem. 2018 Q1(c)]**

Q2. Uma consumes goods 1 and 2. She thinks that 2 units of good 1 is always a perfect substitute for 3 units of good 2. Explain why each of the following utility functions would represent or would not represent Uma's preferences : **[Eco. (H) III Sem. 2019 Q4(b)]**

(i) $U(x_1, x_2) = 3x_1 + 2x_2 + 1000$

(ii) $U(x_1, x_2) = 9x_1^2 + 12x_1x_2 + 4x_2^2$

(iii) $U(x_1, x_2) = \min\{3x_1, 2x_2\}$

(iv) $U(x_1, x_2) = 30x_1 + 20x_2 - 10{,}000$

Q3. Given that utility function for a consumer is given by $U = (x - 3)^2 + (y - 4)^2$, draw any two indifference curves for this consumer while clearly labelling the higher one as 'H' and lower one as 'L'. Does he prefer more of a good to less of it? Does he prefer averages to extreme bundles? Explain. **[Eco. (H) III Sem. 2020 Q1(a)]**

Q4. Check if the following utility functions represent the same preferences :
$u = x + y$, $v = x^3 + y^3$, $w = -1/(x + y)$
Give your reasoning for your answer. **[Eco. (H) III Sem. 2020 Q2(b)]**

Q5. For the following utility functions, discuss whether the indifference curves are downward sloping and convex to the origin or not and also mark the direction of increasing utility on indifference curves. **[Eco. (H) III Sem. 2021 Q1(b)]**

(i) $U(x, y) = x^2(y + 3)$ (ii) $U(x, y) = \left(x^{1/2} + y^{1/2}\right)^2$

(iii) $U(x, y) = (x + 2)(y + 1)$ (iv) $U(x, y) = -3x - 4y$

MA Economics Entrance

Q1. Suppose a consumer's preference are given by the utility function $U = 4\sqrt{x_1} + x_2$. Suppose the consumer's original equilibrium was $x_1 = 9$, $x_2 = 10$. If the consumption of x_1 reduces to 4, how many units of x2 must he consume to remain on the same indifference curve? **[DSE MA Ent. Eco. 2001]**

(a) 10 (b) 8 (c) 14 (d) 5

[**Ans. :** (c)]

Q2. For the same consumer as in question above, the marginal rate of substitution at $x_1 = 9$, $x_2 = 10$ is _____ the marginal rate of substitution at $x_1 = 9$, $x_2 = 20$. **[DSE MA Ent. Eco. 2001]**

(a) Greater than (b) Equal to

(c) Less than (d) Twice

[**Ans. :** (b)]

Q3. For a utility function representing strictly convex preferences, the marginal rate of substitution between two commodities must be **[DSE MA Ent. Eco. 2002]**

(a) Diminishing (b) Increasing

(c) Constant (d) Zero

[**Ans. :** (a)]

Q4. Amit's indifference curves are given by the equation $X_2 = \frac{\text{Constant}}{X_1}$, where larger constants correspond to higher indifference curves. Amit strictly prefers the bundle (7, 15) to the bundle : **[DSE MA Ent. Eco. 2003]**

(a) (15, 7) (b) (8, 14) (c) (11, 11) (d) None of the above

[**Ans. :** (d)]

Q5. Consider Robinson Crusoe who has to allocate 24 hours between lying on the beach and climbing trees to gather coconuts. If L is the time spent on the beach and C is the number of coconuts Crusoe gathers, the Crusoe's utility is min {L, C}. If Crusoe spends $x \leq 8$ hours climbing trees, then he earns x coconuts. If he spends $x > 8$ hours climbing trees, then he earns $8 + 3(x - 8)/2$ coconuts. How many hours should Crusoe spend climbing trees ? **[DSE MA Ent. Eco. 2005]**

(a) 17.4 (b) 12 (c) 11.2 (d) 8

[**Ans. :** (c)]

[**Hint :** $(24 - x) = 8 + 3(x - 8)/2$]

Q6. A consumer has a utility function $U(x, y) = 10(x^2 + 4xy + 4y^2) + 20$. Which one of the following statements must be true? **[DSE MA Ent. Eco. 2008]**

(a) The goods are imperfect substitutes
(b) The goods are perfect substitutes
(c) The goods are perfect complements
(d) None of the above

[**Ans. :** (b)]

Q7. The utility function $u(x, y) = (x + y)^{1/2}$, for $(x, y) \geq (0,0)$ exhibits

(a) Diminishing marginal rate of substitution and diminishing marginal utilities.
(b) Increasing marginal rate of substitution and diminishing marginal utilities.
(c) Constant marginal rate of substitution and diminishing marginal utilities.
(d) Increasing marginal rate of substitution and constant marginal utilities.

[**Ans. :** (c)] **[DSE MA Ent. Eco. 2010]**

Q8. Mr. B thinks cheese is addictive - the more you eat, the more you want. Suppose x denotes the quantity of cheese. Mr. B's utility function can be represented by

(a) $u(x, y) = x^2 + y$ (b) $u(x, y) = \ln x + \ln y$
(c) $u(x, y) = x + y$ (d) $u(x, y) = \min\{x, y\}$

[**Ans. :** (a)] **[DSE MA Ent. Eco. 2012]**