Chapter 2

Mixed Strategy

Learning Objectives :

After learning this chapter you will understand :

- **Mixed Strategies.**
- **Mixed Strategy Nash Equilibrium.**

Basic Concepts

1. **Pure Strategy :** A strategy in which a player makes a specific choice or takes a specific action is called a pure strategy.
2. **Mixed Strategy :** The strategy which is neither selected nor rejected for sure, rather it has some probability to be selected or not is called a mixed strategy. It is a strategy in which a player makes a random choice among two or more possible actions based on set of chosen probabilities is called a mixed strategy.
3. **Mixed Strategy Nash Equilibrium :** A mixed strategy Nash equilibrium involves at least one player playing a randomized strategy, and no player being able to increase their expected payoff by playing an alternate strategy. A Nash equilibrium without randomization is called a pure strategy Nash equilibrium.

   Note that that randomization requires equality of expected payoffs. If a player is supposed to randomize over strategy A or strategy B, then both of these strategies must produce the same expected payoff. Otherwise, the player would prefer one of them, and wouldn't play the other.
4. **Uses of Mixed strategies :** Although at first glance it may seem bizarre to have players finding probabilities of various events (by flipping coins etc.) to determine how they will play, there are good reasons for studying mixed strategies.
   (i) Some games (such as Rock, Paper, Scissors) have no Nash equilibria in pure strategies. Such games will always have a Nash equilibrium in mixed strategies, so allowing for mixed strategies will enable us to make predictions in such games where it was impossible to do so otherwise.
   (ii) Strategies involving randomization are quite natural and familiar in certain settings. Students are familiar with the setting of class exams. Class time is usually too limited for the professor to examine students on every topic taught in class, but it may be sufficient to test students on a subset of topics to induce them to study all of the material. If students knew which topics were on the test then they might be inclined to study only those and not the others, so the professor must choose the topics at random in order to get the students to study everything. Random strategies are also familiar in sports (the same soccer player sometimes shoots to the right of the net and sometimes to the left on penalty kicks) and in card games (the poker player sometimes folds and sometimes bluffs with a similarly poor hand at different times).
   (iii) It is possible to "purify" mixed strategies by specifying a more complicated game in which one or the other action is better for the player for privately known reasons and where that action is played with certainty. For example, a history professor might decide to ask an exam question about World War I because, unbeknownst to the students, she recently read an interesting journal article about it.

5. **Probability Distribution of Mixed Strategies :** If a player has 'M' strategies available to him such as A_1, A_2, A_3, …., A_M and the probability of selecting each

event is σ_1, σ_2, σ_3, …., σ_M respectively, then these actions along with their respective probabilities constitute the probability distribution of mixed strategies.

6. **Strictly Mixed Strategy :** Mixed strategies that involve two or more actions being played with positive probability are called strictly mixed strategies. In the example of mixed strategies in the Battle of the Sexes, all four strategies (1/3, 2/3), (1/2, 1/2), (1, 0), and (0, 1) are mixed strategies. The first two are strictly mixed and the second two are pure strategies.

7. **Existence of Nash Equilibrium :** One of the reasons Nash equilibrium is so widely used is that a Nash equilibrium is guaranteed to exist in a wide class of games. This is not true for some other equilibrium concepts. Consider the dominant strategy equilibrium concept. The Prisoners' Dilemma has a dominant strategy equilibrium (both suspects fink), but most games do not. Indeed, there are many games—including, for example, the Battle of the Sexes—in which no player has a dominant strategy, let alone all the players. In such games, we can't make predictions using dominant strategy equilibrium but we can using Nash equilibrium.

The existence theorem does not guarantee the existence of a pure-strategy Nash equilibrium. However, if a finite game does not have a pure-strategy Nash equilibrium, the theorem guarantees that it will have a mixed-strategy Nash equilibrium.

Exercise 1

Theory Questions

Q1. Define pure strategy and distinguish it from mixed strategy. Show with an example with equilibrium may not exist in pure strategy. Is mixed strategy equilibrium always sensible? **[Eco. (H) 2008]**

Q2. Some games have Nash equilibrium both in pure strategies and mixed strategies. Explain with an example.

Numerical Problems

Q1. Does the Nash equilibrium exist in the following game :

		Person 2	
		Left	Right
Person 1	Top	(1, 0)	(0, 1)
	Bottom	(0.5, 0.5)	(1, 0)

Q2. In the Battle of sexes the payoffs of husband and wife are as under :

		Husband	
		Ballet	Boxing
Wife	Ballet	(2, 1)	(0, 0)
	Boxing	(0, 0)	(1, 2)

(i) Does anybody have a dominant strategy?
(ii) Are there any pure strategy Nash equilibria?
(iii) Is there any mixed strategy Nash equilibrium?
[**Ans. :** (i) No, (ii) (Ballet, Ballet), (Boxing, Boxing), (iii) (2/3, 1/3)]

Q3. In the game of 'Chicken' given below, the two players have the following pay-offs : **[Eco. (H) 2010]**

		Column: Swerve	Column: Stay
Row	Swerve	1, 1	0, 2
	Stay	2, 0	-3, - 3

Does anybody have a dominant strategy? Are there any Nash equilibria in pure strategies? Find a Nash equilibrium in mixed strategies for this game.

Q4. Consider the following game :

		Column	
		Left	Right
Row	Up	(7, 2)	(0, 9)
	Down	(8, 7)	(8, 8)

(i) Does anybody have a dominant strategy?
(ii) Are there any pure strategy Nash equilibria?
(iii) Is there any mixed strategy Nash equilibrium?

Q5. Consider the following game :

		Column	
		Left	Right
Row	Up	(0, 3)	(3, 0)
	Down	(4, 0)	(0, 4)

(i) Does anybody have a dominant strategy?
(ii) Are there any pure strategy Nash equilibria?
(iii) Is there any mixed strategy Nash equilibrium?

Q6. Consider the following game :

		Rocky	
		Party 1	Party 2
You	Party 1	(5, 15)	(20, 10)
	Party 2	(15, 5)	(0, 20)

(i) Show there are no pure strategy Nash equilibria in this game.
(ii) Find the mixed strategy Nash equilibria.
(iii) Show that the probability you encounter Rocky is 7/12.

Q7. Two TV networks are competing for viewer ratings in prime time shows 8-9 pm and 9-10 pm. Each has two shows to fill this time period. Each can choose to put its bigger show first or to place it in second slot. The combination of decision leads to the following rating points results :

		Network 2	
		First	Second
Network 1	First	(15, 15)	(30, 10)
	Second	(20, 30)	(18, 18)

Find the Nash equilibria for this game, assuming that both networks make their decisions at the same time.

Q8. Consider the following game :

		Player II		
		D	E	F
Player 1	A	(7, 6)	(5, 8)	(0, 0)
	B	(5, 8)	(7, 6)	(1, 1)
	C	(0, 0)	(1, 1)	(4, 4)

(a) Find the pure-strategy Nash equilibria (if any).

(b) Find the mixed-strategy Nash equilibrium in which each player randomizes over just the first two actions.

(c) Compute players' expected payoffs in the equilibria found in parts (a) and (b).

(d) Draw the extensive form for this game.

Q9. Two players A and B, play a game where they can play either black or white. If both play black or both play white, B pays A Rs. 100. If A plays black (white) and B plays white (black), no payment is made.

(i) Show the payoff matrix.

(ii) Is there a Nash equilibrium in Pure strategies?

(iii) Find a Nash equilibrium in mixed strategies for this game.

Q10. Consider the game of "Matching Pennies" played by two players, denoted by 1 and 2. Each player simultaneously puts a penny down, either heads up or tails up. if the two pennies match, player 1 pays Re. 1 to player 2; otherwise, player 2 pays Re. 1 to player 1.

(a) Draw normal form of the game.

(b) Find the Nash equilibrium and compute equilibrium payoff.

Q11. Two people can perform a task if and only if they both exert effort. The players preferences or payoffs are represented by the payoff matrix given below :

		Person 2	
		No effort	Effort
Person 1	No effort	(0, 0)	(0, –c)
	Effort	(–c, 0)	(1 – c, 1 – c)

Where, c is the cost of exerting effort and 0 < c < 1. Find all the possible equilibria.

Q12. Assume that the payoff matrix for the Battle of the Sexes is given as under :

		Person 2 (Husband)	
		Ballet	Boxing
Player 1 (Wife)	Ballet	(K, 1)	(0, 0)
	Boxing	(0, 0)	(1, K)

where K ≥ 1. Show how the mixed-strategy Nash equilibrium depends on the value of K.

Q13. Consider the following game :

		Player II		
		Left	Center	Right
Player 1	Top	4, 5	1, 6	5, 6
	Middle	3, 5	2, 5	5, 4
	Bottom	2, 5	2, 0	7, 0

(a) Find the pure-strategy Nash equilibrium(s). Explain the method used.

(b) Find the Mixed Strategy Nash equilibrium(s) in which each player randomizes over just the first two actions; also find and draw their best response functions. **[Eco. (H) 2016]**

Q14. Firm A and B can compete on advertising and R & D. The table below represents the pay-offs measured in profits (Rs. Million) in a one shot simultaneous move game. Firm A's profits are shown first : **[Eco. (H) 2017]**

		Firm B	
		Advertising	R & D
Firm A	Advertising	3, 3	5, 4
	R & D	4, 5	2, 2

(i) Find out all Pure strategy Nash equilibrium of the above game matrix.

(ii) Find the mixed strategy Nash equilibrium, if any.

(iii) Draw the best response function diagram and identify all Nash equilibria.

(iv) What is the expected pay-offs for each firm?

Q15. Two players play a game in which each has three possible strategies, player 1 has to choose from Top, Middle and Bottom, while player 2 chooses from Left, Centre and Right. Their payoffs are given in the table below : **[Eco. (H) 2019]**

		Player 2		
		L	C	R
Player 1	T	3, 2	5, 2	½, 4
	M	4, 3	7, ½	3, 1
	B	5, 2	9, -2	2, 3

(i) Does either player have a strategy that is strictly dominated by another ? If so which one(s)?

(ii) What is/are the pure strategy Nash Equilibria of this game?

(iii) After removing the strategies that are strictly dominated, Find the mixed strategy Nash Equilibrium of the game.

Q16. Consider the following game : **[Eco. (H) 2021]**

		Player 2		
		L	**M**	**R**
Player 1	**U**	8, 8	7, 7	0, 0
	B	7, 7	9, 8	6, 6
	D	0, 0	6, 6	5, 5

(i) Find the pure strategy Nash Equilibrium (if any).

(ii) After eliminating strictly dominating strategies of each player find the mixed strategy Nash Equilibrium.

Q17. Consider the following lobbying game between two firms. Each firm may lobby the government in hopes of persuading it to make a decision that is favourable to the firm. The two firms, F1 and F2, independently and simultaneously decide whether to lobby(L) or not (N). Lobbying entails a cost of 15. Not lobbying costs nothing. If both firms lobby or neither firm lobbies, then the government takes a neutral decision, which yields 10 to both firms. If firm F2 lobbies and F1 does not lobby, then the government makes a decision that favours firm F2, yielding 0 to firm F1 and 30 to firm F2. Finally, If firm F1 lobbies and F2 does not, the government makes a decision in favours of F1, which yields x to firm F1 and zero to firm F2. Assume $x > 25$. The normal form of the game is :

		F2	
		L	N
F1	L	-5, -5	x - 15, 0
	N	0, 15	10, 10

(i) Determine the pure strategy Nash equilibrium of this game.

(ii) Compute the mixed strategy Nash equilibrium of this game.

(iii) Given the mixed strategy Nash equilibrium computed in part (ii), what is the probability that the government makes a decision that favours firm F1?

(iv) As x rises, does the probability that the government makes a decision favouring firm F1 rise or fall? **[Eco. (H) 2022]**

Q18. Consider the following 3 × 3 game : **[Eco. (H) 2023]**

		Player 2		
		L	**C**	**R**
Player 1	**U**	3, 3	2, 1	2, 0
	M	1, 1	1, 1	1, 0
	D	2, 1	4, 2	1, 1

(i) Define Nash Equilibrium and find the pure strategy Nash Equilibrium for this (3 × 3) game.

(ii) Define strictly and weakly dominated actions. Are there any strictly or weakly dominated actions in the given game. Explain your answer.

(iii) Remove the dominated actions and get a 2 × 2 game matrix. Find the Nash equilibrium in a mixed strategy format.

Q19. Consider the following two-player game in which each player has 3 pure strategies **[JNU SIS 2019]**

Player 2

Player 1		L	C	R
	U	4, 3	5, 1	6, 2
	M	2, 1	8, 4	3, 6
	D	3, 0	9, 6	2, 8

Find the strategies that survive iterated elimination of dominated strategies.

(a) (U, L) and (D, C)
(b) (U, L)
(c) (D, C)
(d) None of the strategies survive iterated elimination of strictly dominated strategies

Q20. Consider the following two-player game in which each player has 3 pure strategies

Player 2

Player 1

	L	C	R
U	4, 4	3, 3	0, 0
M	3, 4	5, 4	2, 2
D	0, 0	2, 2	1, 1

Find the strategies that survive iterated elimination of dominated strategies.

Q21. Consider the following two-player game in which each player has 3 pure strategies

Player 2

Player 1

	L	C	R
U	8, 4	3, 3	2, 3
M	3, 1	5, 4	2, 2
D	2, 6	6, 3	2, 4

Find the strategies that survive iterated elimination of dominated strategies.